Answers

Re-Exam 2015

Mathematics for Multimedia Applications
Medialogy

14 August 2015

Formalities

This exam set consists of 4 pages, in which there are 8 problems. You are allowed to use books, notes etc. You are *not* allowed to use electronic devices such as calculators, computers or cell phones.

A number of points is indicated for every sub-problem. The sum of these points equals 100.

Date and time for the exam: 14 August, 9:00 - 13:00.

You must indicate the following on each page:

- Full name
- Study number
- Page number

On the first page, you must indicate

- The total number of pages.

Good luck!

Problems

Problem 1.

1.a. (4 points) Differentiate the function $f(x) = \sin(2x - 1)$. $f'(x) = 2\cos(2x-1)$

1.b. (4 points) Differentiate the function $g(x) = x^3 e^{2x}$. $g'(x) = (3x^2 + 2x^3)e^{2x}$

1.c. (3 points) The graph of the function $g(x)$ above has a tangent at the point $(0, 0)$. What is the slope of that tangent?

$g'(0) = \underline{\underline{0}}$

Problem 2.

$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$.

2.a. (3 points) Prove that the following identity holds: Insert $\alpha = 2x$ and $\beta = x$.

$$\cos(3x) = \cos(2x)\cos(x) - \sin(2x)\sin(x).$$

Hint: Write $3x$ as $2x + x$ and use a trigonometric addition formula.

2.b. (4 points) Prove the following trigonometric identity:

$$\cos(3x) = \cos^3(x) - 3\sin^2(x)\cos(x).$$

Hint: Use the double angle formulas.

Insert
$\cos(2x) = \cos^2(x) - \sin^2(x)$,
$\sin(2x) = 2\sin(x)\cos(x)$
above and reduce
the expression.

2.c. (3 points) Describe all solutions of the equation

$$\cos^3(x) - 3\sin^2(x)\cos(x) = 1.$$

$x = \frac{2\pi}{3}p, \quad p \in \mathbb{Z}$

Problem 3.

3.a. (3 points) Calculate the sum

$$\sum_{i=1}^{5}(i-1)(i+1).$$

50

3.b. (4 points) Calculate the sum

$$\sum_{i=1}^{20}(i^2 - 1).$$

2850

Problem 4.
Evaluate the following integrals:

4.a. (5 points) $\int_0^1 (e^x + x^3)\mathrm{d}x$. $e - \frac{3}{4}$

4.b. (5 points) $\int_0^{\pi/4} (\cos(2x) - \cos(4x))\mathrm{d}x$. $\frac{1}{2}$

Problem 5. Let P, Q, R and S be points in 3D-space with coordinates $(1,0,1)$, $(3,-2,1)$, $(5,-4,1)$ and $(6,-4,0)$ respectively.

5.a. (4 points) Find the coordinates of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{RS}$. Show that the dot product $\overrightarrow{PQ} \bullet \overrightarrow{RS}$ is equal to 2. $\overrightarrow{PQ} = (2,-2,0)$, $\overrightarrow{RS} = (1,0,-1)$,
$\overrightarrow{PQ} \bullet \overrightarrow{RS} = 2 \cdot 1 + (-2) \cdot 0 + 0 \cdot (-1) = 2$.

5.b. (4 points) Let ℓ_1 denote the line through P and Q and ℓ_2 the line through R and S. Find parametric equations of these two lines. $\ell_1 : (x,y,z) = (1,0,1) + t(2,-2,0)$.
$\ell_2 : (x,y,z) = (5,-4,1) + s(1,0,-1)$.

5.c. (3 points) Show that the two lines ℓ_1 and ℓ_2 intersect at the point R.
Insert $t=2$ and $s=0$.

5.d. (4 points) Compute the angle between the lines ℓ_1 and ℓ_2.
60°

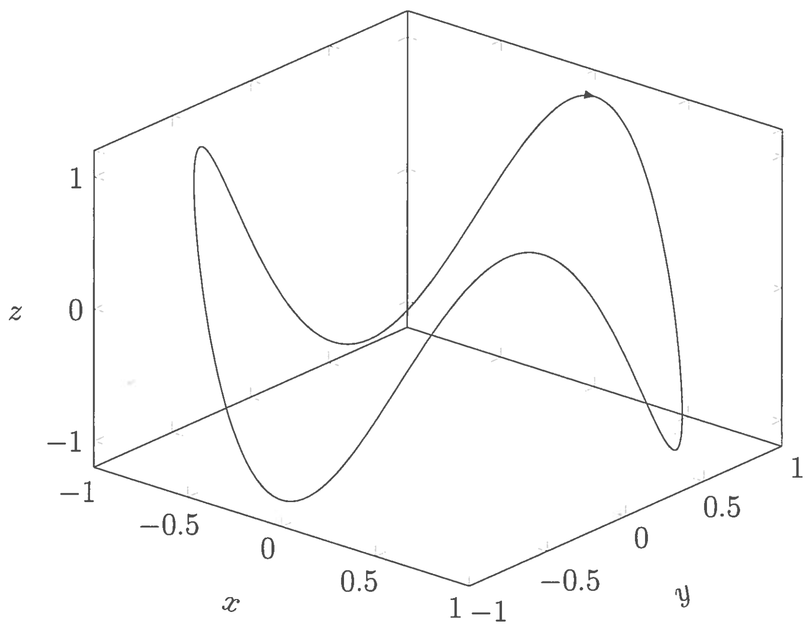
5.e. (3 points) Compute the cross product $\overrightarrow{PQ} \times \overrightarrow{RS}$.

$(2,2,2)$

Problem 6. The position vector of a moving particle in 3D-space is given by

$$\vec{r}(t) = \big(\sin(t), \cos(t), \cos(3t)\big).$$

Here is a plot of the motion curve when the time t runs from 0 to 2π:



6.a. (3 points) Find the velocity vector $\vec{v}(t)$. $\vec{v}(t) = (\cos(t), -\sin(t), -3\sin(3t))$

6.b. (2 points) Find the speed $\nu(t)$. $\nu(t) = \sqrt{1 + 9\sin^2(3t)}$

6.c. (3 points) What is the position vector, velocity vector and speed of the particle at time $t = 0$? $\vec{r}(0) = (0,1,1)$, $\vec{v}(0) = (1,0,0)$, $\nu(0) = 1$

6.d. (3 points) What is the maximal speed of the moving particle? $\sqrt{10}$

6.e. (3 points) Find the acceleration vector $\vec{a}(t)$.

$\vec{a}(t) = (-\sin(t), -\cos(t), -9\cos(3t))$

7.a. $\left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 1 & 1 & 3 & 4 \\ 0 & 2 & 10 & 2 \end{array}\right]$

7.b. $\left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & 5 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Problem 7. Consider the following system of linear equations:

$$\begin{aligned} x_1 - 2x_3 &= 3 \\ x_1 + x_2 + 3x_3 &= 4 \\ 2x_2 + 10x_3 &= 2. \end{aligned}$$

7.a. (3 points) Find the augmented matrix of the system.

7.b. (5 points) Find the reduced row echelon form of the augmented matrix.

7.c. (4 points) Write down the general solution of the system.

$\begin{cases} x_1 = 3 + 2x_3 \\ x_2 = 1 - 5x_3 \\ x_3 \text{ free} \end{cases}$

7.d. (3 points) Find a solution of the system which has $x_2 = 1$.

$x_1 = 3, \quad x_2 = 1, \quad x_3 = 0$

Problem 8. Define two matrices as follows:

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 1 & 0 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 3 \end{bmatrix}.$$

8.a. (4 points) Compute the matrix product AB.

$\begin{bmatrix} 1 & 6 \\ 1 & -4 \\ 1 & 9 \end{bmatrix}$

8.b. (3 points) Find $A + A^T$.

$\begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & -1 \\ 3 & -1 & 6 \end{bmatrix}$

8.c. (4 points) Determine whether A is invertible. If so, find its inverse.

A is invertible

$A^{-1} = \begin{bmatrix} 3 & 0 & -2 \\ -1 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

8.d. (4 points) Solve the following system of linear equations:

$$\begin{aligned} x_1 + 2x_3 &= 2 \\ x_2 - x_3 &= 3 \\ x_1 + 3x_3 &= 1. \end{aligned}$$

$x_1 = 4, \ x_2 = 2, \ x_3 = -1$

Appendix

Exact values for trigonometric functions of various angles.

	0°	30°	45°	60°	90°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0