

Re-Exam 2014

Mathematics for Multimedia Applications
Medialogy

11 August 2014

Formalities

This exam set consists of 5 pages, in which there are 8 problems. You are allowed to use books, notes etc. You are *not* allowed to use electronic devices such as calculators, computers or cell phones.

A number of points is indicated for every sub-problem. The sum of these points equals 100.

Date and time for the exam: 11 August, 9:00 - 13:00.

You must indicate the following on each page:

- Full name
- Study number
- Page number

On the first page, you must indicate

- The total number of pages.

Good luck!

Problems

Problem 1.

1.a. (4 points) Differentiate the function

$$f(x) = \frac{x^5}{e^x}$$

1.b. (4 points) Differentiate the function $g(x) = \cos(x^2 + 1)$.

1.c. (3 points) The graph of the function $g(x)$ above has a tangent at the point $(0, g(0))$. What is the slope of that tangent?

Problem 2.

2.a. (4 points) Prove that the following trigonometric identity holds:

$$\frac{\sin(x) + \cos(x)}{\cos(x)} = \tan(x) + 1$$

2.b. (2 points) Find a solution of the equation

$$\frac{\sin(x) + \cos(x)}{\cos(x)} = 0$$

2.c. (4 points) Describe all solutions of the equation in problem 2.b.

Problem 3.

3.a. (3 points) Calculate the sum

$$\sum_{i=1}^4 (2i - 3)$$

3.b. (5 points) Calculate the sum

$$\sum_{i=1}^9 (4i^3 + 20i)$$

Problem 4. Evaluate the following integrals:

4.a. (5 points) $\int_0^1 (3e^x + 2x)dx$

4.b. (5 points) $\int_0^{\pi/2} (\cos(x) + \cos(2x))dx$

Problem 5. Let P , Q and R be points in 3D-space with coordinates $(5, 4, 6)$, $(7, 4, 4)$ and $(5, 7, 3)$ respectively.

5.a. (3 points) Find \overrightarrow{PQ} and \overrightarrow{PR} .

5.b. (2 points) Find parametric equations for the line through P and Q .

5.c. (3 points) Find the angle between the vectors \overrightarrow{PQ} and \overrightarrow{PR} .

5.d. (3 points) Compute the cross product $\overrightarrow{PQ} \times \overrightarrow{PR}$.

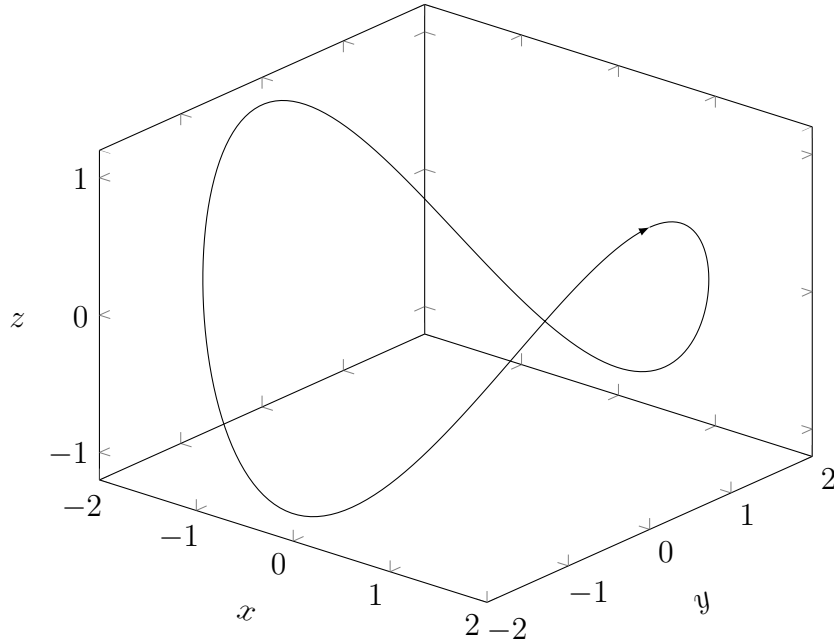
5.e. (3 points) Find $\overrightarrow{PQ} \bullet (\overrightarrow{PQ} \times \overrightarrow{PR})$ and $\overrightarrow{PR} \bullet (\overrightarrow{PQ} \times \overrightarrow{PR})$.

5.f. (3 points) Find an equation for the plane through P , Q and R .

Problem 6. The position vector of a moving particle in 3D-space is given by

$$\vec{r}(t) = (2 \cos(t), 2 \sin(t), \cos(2t))$$

Here is a plot of the motion curve when the time t runs from 0 to 2π :



- 6.a. (4 points) Compute the velocity vector $\vec{v}(t)$.
- 6.b. (3 points) Compute the speed $\nu(t)$.
- 6.c. (3 points) Find the position vector, the velocity vector and the speed of the particle at time $t = 0$.

Problem 7. Consider the following system of linear equations:

$$\begin{aligned} x_1 + x_2 - 3x_3 &= 2 \\ -2x_1 - x_2 + 8x_3 &= -5 \\ x_1 - 5x_3 &= 3 \end{aligned}$$

- 7.a. (3 points) Find the augmented matrix of the system.
- 7.b. (6 points) Find the reduced row echelon form of the augmented matrix.
- 7.c. (4 points) Write down the general solution of the system.
- 7.d. (4 points) Find a solution of the system which has $x_3 = 0$. Find another solution which has $x_3 = 1$.

Problem 8. Define matrices as follows:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & -3 \\ 0 & 1 & 2 \\ 1 & 0 & -5 \end{bmatrix}$$

- 8.a. (3 points) Compute $2A + B^T$.
- 8.b. (4 points) Compute the matrix product AB .
- 8.c. (6 points) Determine whether A is invertible. If so, find its inverse.
- 8.d. (4 points) Solve the following system of linear equations:

$$\begin{aligned}x_1 + x_3 &= 1 \\x_2 + x_3 &= 2 \\x_1 - x_2 - x_3 &= 4\end{aligned}$$

Appendix

Exact values for trigonometric functions of various angles.

	0°	30°	45°	60°	90°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0