Re-Exam 2013

Mathematics for Multimedia Applications
Medialogy

20. August 2013

Formalities

This exam set consists of 6 pages, in which there are 8 problems. You are allowed to use books, notes etc. You are *not* allowed to use electronic devices such as calculators, computers or cell phones.

A number of points is indicated for every sub-problem. The sum of these points equals 100.

Date and time for the exam: 20. August, 9:00 - 13:00.

You must indicate the following on each page:

- Full name
- Study number
- Page number

On the first page, you must indicate

- The total number of pages.

Good luck!

Problems

Problem 1.

1.a. (4 points) Differentiate the function $f(x) = x^4 \ln(x)$.

1.b. (4 points) Differentiate the function $g(x) = \cos(x^3)$.

Problem 2.

2.a. (3 points) Describe all solutions of the equation $\cos(x) = 0$.

2.b. (2 points) Describe all solutions of the equation $\cos^2(x) = 0$.

2.c. (4 points) Prove that the following trigonometric identity holds:

$$(1 + \sin(\theta))(1 - \sin(\theta)) = \cos^2(\theta)$$

Problem 3.

3.a. (3 points) Calculate the sum

$$\sum_{i=1}^{4} (3i - 2)$$

3.b. (5 points) Calculate the sum

$$\sum_{i=1}^{15} (i^2 - 2i)$$

Problem 4. Evaluate the following integrals:

4.a. (4 points) $\int_0^{\pi/6} \cos(3x)\mathrm{d}x$

4.b. (4 points) $\int_0^1 (2e^{2x} + 1)\mathrm{d}x$

Problem 5. Let P, Q and R be points in 3D-space with coordinates $(1, 2, 5)$, $(1, 5, 8)$ and $(-1, 2, 7)$ respectively.

5.a. (3 points) Find $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

5.b. (3 points) Find parametric equations for the line ℓ that passes through the points P and Q.

5.c. (2 points) Compute the dot product $\overrightarrow{PQ} \bullet \overrightarrow{PR}$.

5.d. (3 points) Find the angle between the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

5.e. (3 points) Compute the cross product $\overrightarrow{PQ} \times \overrightarrow{PR}$.

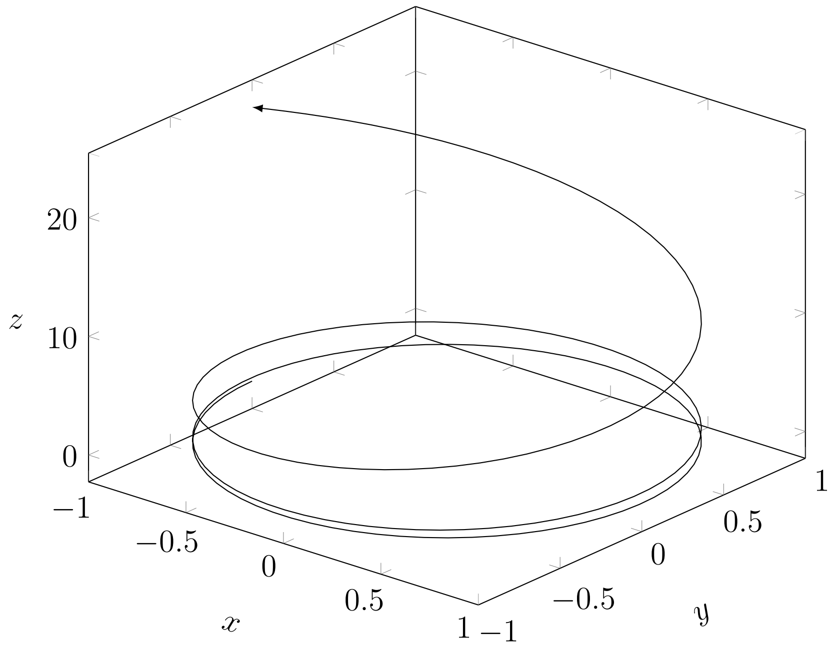
5.f. (3 points) Find the area of the triangle with vertices P, Q and R.

5.g. (4 points) Find the (shortest) distance from the point R to the line ℓ. Hint: Use the area from the question above.

Problem 6. The position vector of a moving particle in 3D-space is given by

$$\vec{r}(t) = \big(\cos(3t), \sin(3t), e^t\big).$$

Here is a plot of the motion curve when the time t runs from $-\pi$ to π:



6.a. (4 points) Compute the velocity vector $\vec{v}(t)$.

6.b. (3 points) Compute the speed $\nu(t)$.

6.c. (2 points) Find the position of the particle at time $t = 0$.

6.d. (2 points) What is the speed of the moving particle at time $t = 0$?

6.e. (4 points) In which direction is the particle moving at time $t = 0$? Find a unit vector describing the direction.

Problem 7. Consider the following system of linear equations:

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 7 \\ 2x_1 + 6x_2 &= 4 \\ x_1 + 3x_2 - x_3 &= -3 \end{aligned}$$

7.a. (2 points) Is $x_1 = 2$, $x_2 = 0$, $x_3 = 5$ a solution of the system? Why/why not?

7.b. (3 points) Find the augmented matrix of the system.

7.c. (5 points) Find a row echelon form of the augmented matrix.

7.d. (3 points) Find the reduced row echelon form of the augmented matrix.

7.e. (4 points) Write down the general solution of the system.

7.f. (3 points) Find a solution of the system which has $x_1 = -1$.

Problem 8. Define three matrices as follows:

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 3 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

8.a. (3 points) Compute $A + B^T$.

8.b. (4 points) Compute the matrix product AB.

8.c. (4 points) Determine whether C is invertible. If so, find its inverse.

Appendix

Exact values for trigonometric functions of various angles.

	30°	45°	60°
	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$