Re-Exam 2012

Mathematics for Multimedia Applications
AAU-Cph, Medialogy

16. August 2012

Formalities

This re-exam set consists of 5 pages, in which there are 9 problems in total. You are allowed to use books, notes etc. You are *not* allowed to use electronic devices such as calculators, computers or cell phones.

A number of points is indicated for every sub-problem. The sum of these points equals 100.

Date and time for the exam: 16. August, 9:00 - 13:00

You must indicate the following on each page:

- Full name
- Study number
- Page number

On the first page, you must indicate

- The total number of pages.

Good luck!

Problems

Problem 1.

1.a. (3 points) Differentiate the function $f(x) = \sin(3x) + \cos(2x)$.

1.b. (3 points) Differentiate the function $g(x) = x^3 e^{5x}$.

1.c. (3 points) Differentiate the function $h(x) = e^{x^2+1}$

Problem 2. (5 points)

Prove that the following trigonometric identity holds:

$$\cos(\alpha)\sin(\beta) = \frac{1}{2}(\sin(\alpha+\beta) - \sin(\alpha-\beta)).$$

Hint: Start with the right hand side of the equation. Use the trigonometric addition formulas.

Problem 3.

3.a. (3 points) Calculate the sum

$$\sum_{i=1}^{5} i(i+1).$$

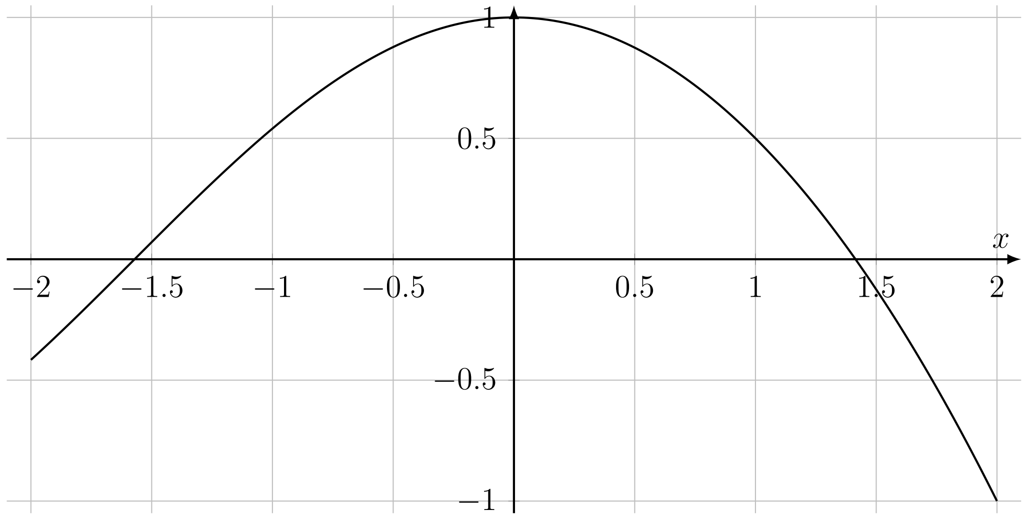
3.b. (4 points) Calculate the sum

$$\sum_{i=1}^{10} (4i^3 + 2).$$

Problem 4. Let f be the function defined by

$$f(x) = \begin{cases} \cos(x), & x \leq 0, \\ 1 - \frac{1}{2}x^2, & x > 0. \end{cases}$$

The graph of f looks as follows:



4.a. (3 points) Evaluate the integral $\int_{-\pi/2}^{0} f(x)\mathrm{d}x$.

4.b. (3 points) Evaluate the integral $\int_{0}^{1} f(x)\mathrm{d}x$.

4.c. (3 points) Find $\int_{-\pi/2}^{1} f(x)\mathrm{d}x$.

Problem 5. Let P, Q and R be three points in 3D-space. P has coordinates $(4,2,2)$, Q has coordinates $(4,1,3)$ and R has coordinates $(5,2,3)$.

5.a. (2 points) Find $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

5.b. (2 points) Find parametric equations for the line that passes through P and Q.

5.c. (2 points) Compute the dot product $\overrightarrow{PQ} \bullet \overrightarrow{PR}$.

5.d. (3 points) Compute the cross product $\overrightarrow{PQ} \times \overrightarrow{PR}$.

5.e. (3 points) Find the angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

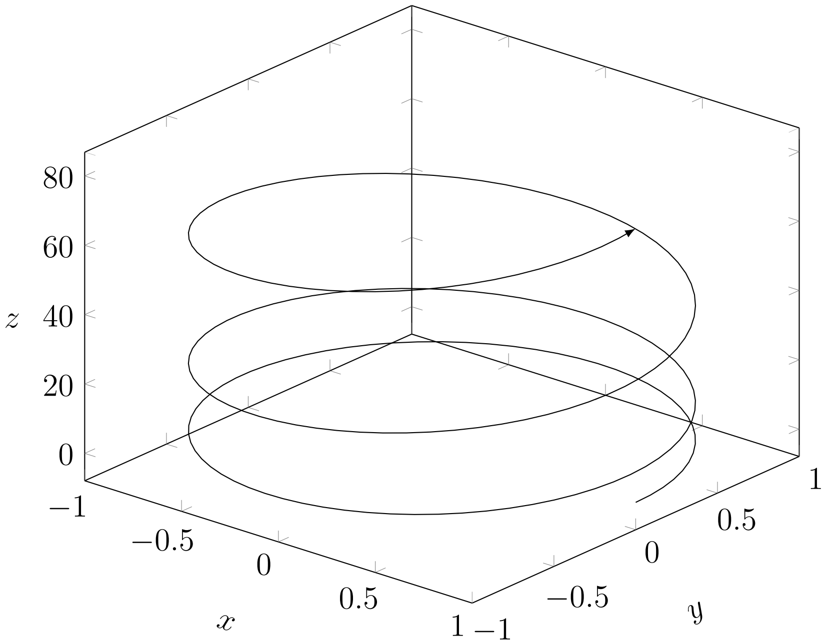
5.f. (3 points) Find the area of the triangle with vertices P, Q and R.

5.g. (3 points) Find an equation for the plane through P, Q and R.

Problem 6. A parametric curve is given by the following vector function:

$$\vec{r}(t) = (\cos(3t), \sin(3t), 2t^2).$$

Here is a plot of the curve when t runs from 0 to 2π:



6.a. (3 points) Compute the velocity vector $\vec{v}(t)$.

6.b. (3 points) Compute the speed $\nu(t)$.

6.c. (3 points) Find a $t > 0$ such that $\nu(t) = 5$.

Problem 7. Consider the following system of linear equations:

$$\begin{aligned} x_1 + 2x_2 + x_3 + 2x_4 &= 4 \\ 2x_1 + 4x_2 + x_3 + x_4 &= 9 \\ 3x_1 + 6x_2 + 2x_3 + 3x_4 &= 13 \end{aligned}$$

7.a. (2 points) Is $x_1 = 5$, $x_2 = 1$, $x_3 = -7$, $x_4 = 2$ a solution to the system? Why/why not?

7.b. (2 points) Find the augmented matrix of the system.

7.c. (5 points) Find the reduced row echelon form of the augmented matrix.

7.d. (4 points) Write down the general solution to the system.

7.e. (3 points) Find a solution to the system which has $x_2 = 1$ and $x_3 = -4$.

Problem 8. Define three matrices as follows:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 5 & -1 \\ 2 & 7 & -3 \\ 1 & -1 & -3 \end{bmatrix}$$

8.a. (2 points) Compute $A + B^T$.

8.b. (3 points) Compute the matrix product AB.

8.c. (3 points) Show that A is invertible and compute A^{-1}.

8.d. (3 points) Show that B is invertible and compute B^{-1}.

8.e. (3 points) Find $(AB)^{-1}$.

8.f. (4 points) Is C an invertible matrix? Why/ why not?

Problem 9. Consider an (x, y)-coordinate system in the plane.

9.a. (3 points) Write down the rotation matrix R for a 30° counterclockwise rotation about the origin.

9.b. (3 points) Let P be the point with coordinates $(2, 6)$. Apply the above rotation on P. What are the coordinates for the rotated point?

9.c. (3 points) Find the inverse matrix R^{-1}.