Midterm Reexam in Mathematics for Multimedia Applications

First Year at the Technical Faculty of IT and Design

7 August 2018, 10:00-11:00

This exam set consists of 3 pages with 6 problems. A number of points is indicated for each question. The total number of points equals 25.

It is allowed to use books, notes, photocopies etc. It is **not allowed** to use **any electronic devices** such as pocket calculators, mobile phones or computers.

The exam set has two independent parts.

- Part I contains "multiple choice" problems. **The answers of Part I must be given on these sheets.**
- Part II contains an "essay problem". Here it is important that you explain the idea behind the solution, and that you provide relevant intermediate results.

Remember to write your full name (including middle names) together with your student number below. Also write name and student number on each page of your solution of the essay problem and number these pages. Indicate the total number of extra sheets on the first page.

Good luck!

NAME: ______________________________

STUDENT NUMBER: ______________________________

Answers

Part I (Multiple-choice problems)

Problem 1 (1 point)

The size of an angle is $\frac{\pi}{20}$ radians. What is the size of the angle measured in degrees?

☐ 20° ☐ 15° ☐ 12° ☑ 9° ☐ 5° ☐ 2°

Problem 2 (4 points)

A particle is moving along a horizontal axis. Its position as a function of time is given by

$$x(t) = e^t - t + 2.$$

What is the position of the particle when its velocity is equal to 0?

☐ -2 ☑ 3 ☐ $\ln(2)$ ☐ e^3 ☐ e ☐ $e - 1$

Problem 3 (3 points)

A function is given by

$$f(x) = \cos(-3x^2 + x).$$

What is the derivative $f'(x)$?

☐ $\cos(-6x + 1)$ ☐ $\cos(-\frac{x^3}{3} + \frac{x^2}{2})$

☐ $-\sin(-6x + 1)$ ☑ $(6x - 1)\sin(-3x^2 + x)$

☐ $\sin(2x + 2)$ ☐ $-\sin(-3x^2 + x)$

Problem 4 (4 points)

A function is defined by

$$g(x) = \ln(2x).$$

The tangent slope equals 2 at a point on the graph of g. What are the coordinates of that point?

☐ $(1, \ln(2))$ ☐ $(\frac{1}{4}, -\ln(2))$ ☑ $(\frac{1}{2}, 0)$

☐ $(3, \ln(6))$ ☐ $(\frac{e}{2}, 1)$ ☐ $(\frac{e}{4}, 1 - \ln(2))$

Problem 5 (4 points)

A curve is given by the equation

$$y = 2\sin(x) + 5.$$

What is the largest slope that a line tangent to this curve can have?

☐ -1 ☐ 0 ☐ 1 ☑ 2 ☐ 4 ☐ 5

Part II (Essay-problem)

Problem 6 (9 points)

(a) (3 points). Show that the solutions of the equation

$$\cos(x)\sin(x) = 0$$

can be described as $x = \frac{\pi p}{2}$ where p runs through the integers.
Hint: Use the unit circle.

(b) (3 points). Prove that the following trigonometric identity holds:

$$(\cos(x) + \sin(x))^2 = 1 + 2\cos(x)\sin(x).$$

(c) (3 points). Describe all solutions of the equation

$$\cos(x) + \sin(x) = 1.$$

(a) One has

$$\cos(x)\sin(x) = 0 \Leftrightarrow \cos(x) = 0 \vee \sin(x) = 0.$$

The solutions correspond to the four points $(1,0)$, $(0,1)$, $(-1,0)$ and $(0,-1)$ where the unit circle intersects the coordinate axis. The direction angles of these points are

$$x = \frac{\pi p}{2}, \quad p \in \mathbb{Z}.$$

(b)

$$\begin{aligned}(\cos(x) + \sin(x))^2 &= \cos^2(x) + \sin^2(x) + 2\cos(x)\sin(x)\\ &= 1 + 2\cos(x)\sin(x).\end{aligned}$$

(c) By (b) the solutions of this equation will also be solutions of the equation in (a). However, only the points $(1,0)$ and $(0,1)$ represents solutions of the equation in (c). The corresponding direction angles are

$$x = 2\pi p \vee x = \frac{\pi}{2} + 2\pi p, \quad p \in \mathbb{Z}.$$