

# Exam in Mathematics for Multimedia Applications

First Year at the Technical Faculty of IT and Design

13 August 2019, 9:00-13:00

This exam set consists of 6 pages with 12 problems. A number of points are specified for each question. The total number of points equals 100.

It is allowed to use books, notes, photocopies etc. It is **not allowed** to use **any electronic devices** such as pocket calculators, mobile phones, or computers.

The exam set has two independent parts.

- Part I contains “essay problems”. Here it is important that you explain the idea behind the solution, and that you provide relevant intermediate results.
- Part II contains “multiple choice” problems. **The answers of Part II must be given on these sheets.**

Remember to write your full name (including middle names) together with your student number below. Also write name and student number on each page of your solutions of the essay problems and number these pages. Indicate the total number of extra sheets on the first page.

Good luck!

NAME: \_\_\_\_\_

STUDENT NUMBER: \_\_\_\_\_

## Part I (Essay problems)

### Problem 1 (9 points)

(a) (5 points). Prove that the following trigonometric identity holds:

$$\frac{\sin(x) + \cos(x)}{\cos(x) - \sin(x)} = \tan\left(x + \frac{\pi}{4}\right).$$

(b) (4 points). Describe all solutions of the equation

$$\frac{\cos(x) + \sin(x)}{\cos(x) - \sin(x)} = \sqrt{3}.$$

### Problem 2 (19 points)

Consider the system of linear equations

$$\begin{aligned}x_1 + 2x_3 + x_4 &= 1, \\x_1 + 2x_2 + 3x_3 &= 1, \\2x_1 + x_2 + 4x_3 &= 1.\end{aligned}\tag{1}$$

(a) (2 points). Write down the augmented matrix corresponding to the system of linear equations (1).

(b) (10 points). Compute the reduced row echelon form of the augmented matrix corresponding to the system (1).

(c) (4 points). Find all solutions (if any) to the system (1).

(d) (3 points). Find all solutions (if any) to the system

$$\begin{aligned}x_1 + 2x_3 + x_4 &= 1, \\x_1 + 2x_2 + 3x_3 &= 1, \\2x_1 + x_2 + 4x_3 &= 1, \\x_1 - 5x_4 &= 2019.\end{aligned}$$

## Part II (Multiple choice problems)

### Problem 3 (6 points)

Let  $a$  be a real constant. Mark the correct expression for the limit

$$\lim_{h \rightarrow 0} \frac{2 \sin((a+h)^3) - 2 \sin(a^3)}{h}.$$

$6 \sin(a^3)$

$6a^2 \cos(a^3)$

$2 \sin(a^3)$

$2 \cos(a^3)$

$6a^2 \sin(a^3)$

$2 \tan(a^3)$

#### Problem 4 (6 points)

A function is given by

$$f(x) = 3x(\ln(x) - 1).$$

Mark the correct expression for its derivative  $f'(x)$ .

$\ln(x^3)$

$\frac{3 - \ln(x^3)}{x^2}$

$\frac{3}{x}$

$\frac{1 - 3 \ln(x)}{x^2}$

$\frac{3\frac{1}{x} - 3}{x^2}$

$3e^x$

#### Problem 5 (6 points)

A function is defined by

$$g(x) = \exp(6x^2 - 3x + 19).$$

The graph of the function has a horizontal tangent at a point. What is the  $x$ -coordinate of that point?

$1/4$

$3$

$1$

$4$

$-2$

$0$

#### Problem 6 (6 points)

A particle is moving along a horizontal  $x$ -axis. Its velocity as a function of time is given by  $v(t) = 6te^t + 2t$ . At time  $t = 0$ , the particle is located at  $x = -6$ . What is the position function for the particle?

$x(t) = e^{6t} + t^2 - 7$

$x(t) = 6te^t + 2t + 4$

$x(t) = 6te^{6t} + 2t - 6$

$x(t) = 6e^t + 2t - 2$

$x(t) = 6te^t + t^2 + 4$

$x(t) = 6te^t + t^2 - 6e^t$

#### Problem 7 (4 points)

The integral

$$\int_{-100}^{100} x^3 dx$$

is equal to

- 0                       2000000                       -10000000  
 25000000                       10000000                       50000000

### Problem 8 (5 points)

Mark the value of the sum

$$\sum_{i=0}^4 (i^2 - 5).$$

- 30                       5                       0  
 55                       10                       -30

### Problem 9 (6 points)

Mark the value of the sum

$$\sum_{i=1}^{10} 3(i^3 - i).$$

- 8200                       5200                       8910  
 9020                       8120                       6400

### Problem 10 (11 points)

Four points in 3D-space are given by

$$P = (-1, 0, 0), \quad Q = (0, -1, 0), \quad R = (3, 2, 2), \quad S = (2, 0, 1).$$

In consequence, we have the following two vectors

$$\vec{PQ} = (1, -1, 0), \quad \vec{RS} = (-1, -2, -1).$$

Mark the correct answers below.

(a) (2 points). The coordinates of the vector  $\vec{PR}$  are

- (1, -1, 2)                       (4, 2, 4)  
 (4, 2, 2)                       (3, -1, -4)

(b) (3 points). The line  $\ell_1$  through  $P$  and  $Q$  has parametric equation

- $(x, y, z) = (-1, 0, 0) + t(1, -1, 0)$   
  $(x, y, z) = (-3, 1, 1) + t(1, -1, 0)$   
  $(x, y, z) = (-2, 0, 1) + t(-4, -1, 1)$   
  $(x, y, z) = (-3, 1, 1) + t(-2, 0, 1)$



### Problem 12 (12 points)

Decide whether the following statements are true or false.

- (a) (2 points). For all  $n \times n$ -matrices  $A$  and  $B$  and real numbers  $c$ , it holds

$$c(AB)^T = B^T(cA)^T.$$

True

False

- (b) (2 points). For all invertible  $n \times n$ -matrices  $A$  and  $B$  and real numbers  $c$ , it holds

$$c(AB)^{-1} = B^{-1}(cA)^{-1}.$$

True

False

- (c) (2 points). Let  $R_\phi$  and  $R_\theta$  be  $2 \times 2$ -rotation matrices corresponding to angles  $\phi$  and  $\theta$ , respectively. Then it holds  $R_\phi R_\theta^{-1} = R_{\phi-\theta}$ .

True

False

- (d) (2 points). Let  $R_\phi$  be a  $2 \times 2$ -rotation matrix corresponding to the angle  $\phi$ . Then it holds  $R_\phi^{-1} = R_\phi^T$ .

True

False

- (e) (2 points). The following matrix represents a rotation about the angle  $3\pi/2$  in the counter-clockwise direction:

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

True

False

- (f) (2 points). Let  $a, b, c, d$  be real numbers with  $ad - bc = 1$ . Then the inverse of the matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

is given by

$$\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

True

False