Reexam in Mathematics for Multimedia Applications

First Year at the Technical Faculty of IT and Design

14 August 2018, 9:00-13:00

This exam set consists of 7 pages with 9 problems. A number of points is indicated for each question. The total number of points equals 75.

It is allowed to use books, notes, photocopies etc. It is **not allowed** to use **any electronic devices** such as pocket calculators, mobile phones or computers.

The exam set has two independent parts.

- Part I contains an "essay problem". Here it is important that you explain the idea behind the solution, and that you provide relevant intermediate results.
- Part II contains "multiple choice" problems. **The answers of Part II must be given on these sheets.**

Remember to write your full name (including middle names) together with your student number below. Also write name and student number on each page of your solution of the essay problem and number these pages. Indicate the total number of extra sheets on the first page.

Good luck!

NAME: ____________________________________

STUDENT NUMBER: ____________________________________

Part I (Essay-problem)

Problem 1 (12 points)

A system of linear equations is given by

$$\begin{aligned} x_1 + x_2 &= 2 \\ x_1 + 2x_3 &= -1 \\ 2x_1 - x_2 + 6x_3 &= -5 \\ 2x_1 + x_2 + 2x_3 &= 1. \end{aligned}$$

(a) (3 points). Find the augmented matrix of the system.

(b) (5 points). Find the reduced row echelon form of the augmented matrix.

(c) (4 points). Write down the general solution of the system.

Part II (Multiple-choice problems)

Problem 2 (6 points)

A matrix is given by

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}.$$

Mark the correct statement below.

☐ A is invertible and entry $(3,4)$ of its inverse, i.e. $[A^{-1}]_{34}$, equals 0.

☐ A is invertible and entry $(3,4)$ of its inverse, i.e. $[A^{-1}]_{34}$, equals -1.

☐ A is invertible and entry $(3,4)$ of its inverse, i.e. $[A^{-1}]_{34}$, equals 1.

☐ A is invertible and entry $(3,4)$ of its inverse, i.e. $[A^{-1}]_{34}$, equals 3.

☐ A is not invertible.

☐ None of the above statements apply.

Problem 3 (9 points)

Three matrices are given by

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 3 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 2 & 3 \\ 1 & 0 & 5 \\ 1 & -1 & 2 \end{bmatrix}.$$

Mark the correct statements below.

(a) (3 points). The matrix product ABC has size

☐ 3×4 ☐ 2×3 ☐ 3×3 ☐ 4×3

(b) (3 points). Entry $(3, 4)$ of the matrix product AB, i.e. $[AB]_{34}$, equals

☐ 2 ☐ 0 ☐ 9 ☐ -3

☐ -1 ☐ 3 ☐ 12 ☐ 1

(c) (3 points). Entry $(4, 3)$ of the matrix $B^T A^T$ i.e. $[B^T A^T]_{43}$, equals

☐ 0 ☐ 7 ☐ -3 ☐ 2

☐ -2 ☐ 5 ☐ 1 ☐ 3

Problem 4 (3 points)

Mark the value of the sum

$$\sum_{i=1}^{6} (i^2 - i - 3).$$

☐ 45 ☐ 60 ☐ 31 ☐ 51

☐ 49 ☐ 72 ☐ 39 ☐ 52

Problem 5 (6 points)

Mark the value of the sum

$$\sum_{i=1}^{24} \frac{i}{3}.$$

☐ 52	☐ 100	☐ 210	☐ 85
☐ 55	☐ 110	☐ 220	☐ 90

Problem 6 (8 points)

Evaluate the integrals below and mark the correct results.

(a) (4 points). The integral

$$\int_0^1 (x^5 + e^{6x})dx$$

is equal to

☐ $\frac{e^6}{6}$	☐ e^6	☐ $\frac{1}{7}$	☐ $\frac{e^7}{7}$
☐ $\frac{e^7}{7} - 1$	☐ $\frac{1}{6}$	☐ $\frac{13}{42}$	☐ $\frac{e^6}{6} - 1$

(b) (4 points). The integral

$$\int_0^{\pi/4} \sin(8x)dx$$

is equal to

☐ 1	☐ 0	☐ 2π	☐ $-\frac{1}{8}$
☐ $\frac{1}{8}$	☐ π	☐ -1	☐ $-\pi$

Problem 7 (17 points)

Three points in 3D-space are given by

$$P = (4,3,2), \quad Q = (5,2,1), \quad R = (5,4,4).$$

In consequence, we have the following vectors

$$\overrightarrow{PQ} = (1,-1,-1), \quad \overrightarrow{PR} = (1,1,2).$$

Mark the correct answers below.

(a) (2 points). The coordinates of the vector $\overrightarrow{QR}$ are

☐ $(0,2,-3)$	☐ $(0,1,6)$	☐ $(1,0,3)$
☐ $(1,-1,2)$	☐ $(0,2,3)$	☐ $(10,6,5)$

(b) (3 points). The cross product $\overrightarrow{PQ} \times \overrightarrow{PR}$ equals

☐ $(1,10,3)$	☐ $(-1,-3,2)$	☐ $(5,2,-1)$
☐ $(4,-4,3)$	☐ $(1,5,2)$	☐ $(4,-1,2)$

(c) (3 points). The area of the triangle with vertices P, Q and R is

☐ $\sqrt{19}$	☐ $\frac{5}{2}$	☐ $\frac{\sqrt{14}}{2}$
☐ $\frac{\sqrt{2}}{2}$	☐ $\sqrt{15}$	☐ $\frac{\sqrt{10}}{2}$

(d) (3 points). The line through P and Q has parametric equation

☐ $(x,y,z) = (5,4,4) + t(1,1,1)$	☐ $(x,y,z) = (4,3,2) + t(5,2,1)$
☐ $(x,y,z) = (1,2,1) + t(1,0,2)$	☐ $(x,y,z) = (5,2,1) + t(4,3,2)$
☐ $(x,y,z) = (1,-1,-1) + t(5,1,4)$	☐ $(x,y,z) = (4,3,2) + t(1,-1,-1)$

(e) (3 points). The angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ is

☐ $\cos^{-1}(-\frac{\sqrt{2}}{3})$	☐ $\cos^{-1}(\frac{1}{3})$	☐ $\frac{\pi}{3}$
☐ $\cos^{-1}(\frac{\sqrt{5}}{6})$	☐ $\cos^{-1}(-\frac{1}{4})$	☐ $\frac{\pi}{4}$

(f) (3 points). Which one of the following points belongs to the plane through P, Q and R?

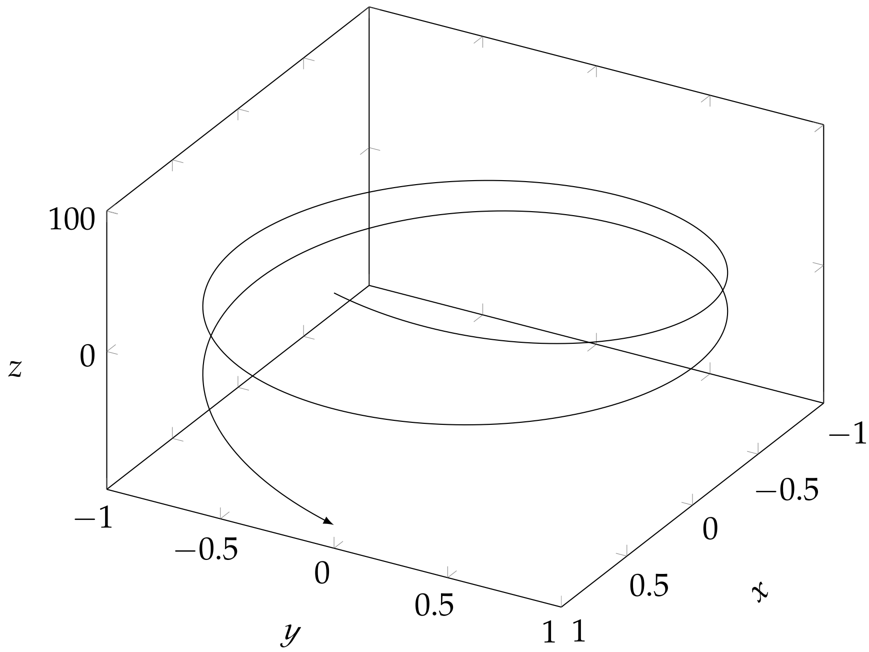
☐ $(1,2,1)$	☐ $(3,3,1)$	☐ $(1,1,-2)$
☐ $(1,1,1)$	☐ $(0,4,1)$	☐ $(0,3,0)$

Problem 8 (8 points)

The position vector of a moving particle in 3D-space is given by

$$\vec{r}(t) = \big(\cos(t), \sin(t), 1 - \tfrac{1}{3}t^3\big).$$

Here is a plot of the motion curve when the time t runs from -2π to 2π:



(a) (2 points). What are the coordinates of the particle at time $t = 0$?

☐ $(0, 0, 1)$	☐ $(1, 0, \frac{2}{3})$
☐ $(1, -1, 1)$	☐ $(1, 1, \frac{1}{3})$
☐ $(1, 0, 0)$	☐ $(1, 0, 1)$

(b) (3 points). What is the velocity vector for the particle at time $t = 0$?

☐ $(1, 0, 0)$	☐ $(0, 1, 1)$
☐ $(1, 0, 1)$	☐ $(0, 0, 1)$
☐ $(0, 1, 0)$	☐ $(1, 0, \frac{2}{3})$

(c) (3 points). What is the speed $v(t)$ of the moving particle?

☐ $\sqrt{1 + t^6}$	☐ $\sqrt{1 + t^4}$
☐ $\sqrt{\cos(t) - \sin(t) - t^2}$	☐ $\sqrt{3}t$
☐ $\sqrt{2}$	☐ $1 + t^2$

Problem 9 (6 points)

Consider the rotation matrix R_θ and the vector $\vec{v}$ defined as

$$R_\theta = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

1. (a) (3 points). What is the matrix-vector product $R_{\pi/4}\vec{v}$?

☐ $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ ☐ $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ ☐ $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

☐ $\begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ☐ $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$

2. (b) (3 points). What is the matrix-vector product $(R_{\pi/4})^2\vec{v}$?

☐ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ ☐ $\begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}$ ☐ $\begin{bmatrix} \sqrt{2} \\ \sqrt{2} \end{bmatrix}$

☐ $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$ ☐ $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ ☐ $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$