Rexam in Mathematics for Multimedia Applications

First Year at the Technical Faculty of IT and Design

22 August 2017

This exam set consists of 8 pages with 14 problems. A number of points is indicated for each question. The total number of points equals 100.

It is allowed to use books, notes, photocopies etc. It is **not allowed** to use **any electronic devices** such as pocket calculators, mobile phones or computers.

The exam set has two independent parts.

- Part I contains "essay problems". Here it is important that you explain the idea behind the solution, and that you provide relevant intermediate results.
- Part II contains "multiple choice" problems. **The answers of Part II must be given on these sheets.**

Remember to write your full name (including middle names) together with your student number below. Also write name and student number on each page of your solutions of the essay problems and number these pages. Indicate the total number of extra sheets on the first page.

Good luck!

NAME: ______________________________

STUDENT NUMBER: ______________________________

Answers

Part I (Essay-problems)

Problem 1 (8 points)

(a) (5 points). Prove that the following trigonometric identity holds:

$$\frac{\sin(2x)}{\cos(2x)+1} = \tan(x).$$

(b) (3 points). Describe all solutions of the equation

$$\frac{\sin(2x)}{\cos(2x)+1} = 1.$$

(a) Use the double angle formulas for sine and cosine and simplify the expression.

(b) $x = \frac{\pi}{4} + \pi p, \quad p \in \mathbb{Z}.$

Problem 2 (12 points)

A system of linear equations is given by

$$\begin{aligned} x_1 + 7x_2 + 3x_3 + 5x_4 &= 4 \\ 2x_1 + 14x_2 + 3x_3 + 4x_4 &= 5 \\ x_1 + 7x_2 - x_3 - 3x_4 &= 0. \end{aligned}$$

(a) (3 points). Find the augmented matrix of the system.

(b) (5 points). Find the reduced row echelon form of the augmented matrix.

(c) (4 points). Write down the general solution of the system.

$$\begin{bmatrix} 1 & 7 & 3 & 5 & 4 \\ 2 & 14 & 3 & 4 & 5 \\ 1 & 7 & -1 & -3 & 0 \end{bmatrix}, \qquad \begin{bmatrix} 1 & 7 & 0 & -1 & 1 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \begin{aligned} &x_1 = 1 - 7x_2 + x_4 \\ &x_2 \text{ free} \\ &x_3 = 1 - 2x_4 \\ &x_4 \text{ free} \end{aligned}$$

Part II (Multiple-choice problems)

Problem 3 (5 points)

Let a be a constant such that $-\frac{\pi}{2} < a < \frac{\pi}{2}$. Mark the correct expression for the limit

$$\lim_{h\to 0} \frac{\tan(a+h) - \tan(a)}{h}.$$

- [ ] $2\tan a$
- [ ] 1
- [x] $1+\tan^2 a$
- [ ] $\cos^2 a$
- [ ] 0
- [ ] ∞

Problem 4 (4 points)

A function is given by

$$f(x) = \frac{x^3}{e^x}.$$

Mark the correct expression for its derivative $f'(x)$.

- [ ] $\frac{3x^2}{e^x}$
- [ ] $3x^2 \ln x$
- [ ] $\frac{x^4}{4e^x}$
- [ ] $x^3 \ln x + 3x^2 e^x$
- [ ] $2xe^{-x}$
- [x] $\frac{3x^2 - x^3}{e^x}$

Problem 5 (6 points)

A function is defined by

$$g(x) = e^{x^2+6x+1}.$$

The graph of the function has a horizontal tangent at a point. What is the x-coordinate of that point?

- [x] -3
- [ ] 3
- [ ] 1
- [ ] 5
- [ ] -2
- [ ] 0

Problem 6 (4 points)

A particle is moving along a horizontal x-axis. Its velocity as a function of time is given by $v(t) = 6t + 2$. At time $t = 1$ the particle is located at $x = 4$. What is the position function for the particle?

☐ $x(t) = 2t^2 - t + 3$	☑ $x(t) = 3t^2 + 2t - 1$	☐ $x(t) = t^2 - 2t + 5$
☐ $x(t) = t^2 + 6t - 3$	☐ $x(t) = t^3 - t + 4$	☐ $x(t) = \sin(3t^2) + 2t$

Problem 7 (4 points)

The integral

$$\int_0^{\pi/2} (\cos(x) + \cos(-x))dx$$

is equal to

☐ 0	☐ π	☑ 2
☐ 1	☐ -1	☐ 2π

Problem 8 (3 points)

Mark the value of the sum

$$\sum_{i=1}^{5} (2i^2 - 5i).$$

☐ 31	☐ 11	☑ 35
☐ 40	☐ 19	☐ 29

Problem 9 (6 points)

Mark the value of the sum

$$\sum_{i=1}^{9} 4(i^3 + i).$$

☐ 9100	☐ 12100	☐ 9180
☑ 8280	☐ 8890	☐ 10900

Problem 10 (14 points)

Four points in 3D-space are given by

$$P = (-1, 1, 3), \quad Q = (0, 1, 2), \quad R = (-3, 5, -1), \quad S = (-1, 3, 0).$$

In consequence, we have the following two vectors

$$\overrightarrow{PQ} = (1, 0, -1), \quad \overrightarrow{RS} = (2, -2, 1).$$

Mark the correct answers below.

(a) (2 points). The coordinates of the vector $\overrightarrow{PR}$ are

- ☐ $(1, 2, -1)$
- ☐ $(1, 1, -3)$
- ☑ $(-2, 4, -4)$
- ☐ $(1, -4, -3)$

(b) (3 points). The line ℓ_1 through P and Q has parametric equation

- ☐ $(x, y, z) = (-1, 3, 0) + t(1, 2, -1)$
- ☑ $(x, y, z) = (-1, 1, 3) + t(1, 0, -1)$
- ☐ $(x, y, z) = (0, 1, 2) + t(1, 1, 4)$
- ☐ $(x, y, z) = (-1, 1, 3) + t(0, 1, 2)$

(c) (3 points). One finds, that the line ℓ_2 through R and S, has parametric equation

$$(x, y, z) = (-3, 5, -1) + s(2, -2, 1).$$

The two lines ℓ_1 and ℓ_2 intersect at the point $T = (1, 1, 1)$. Which values of the parameters t and s correspond to this intersection point?

- ☑ $t = 2, s = 2$
- ☐ $t = 2, s = -1$
- ☐ $t = 1, s = 3$
- ☐ $t = 2, s = 0$

(d) (3 points) What is the angle between the two lines ℓ_1 and ℓ_2?

- ☐ $\frac{\pi}{2}$
- ☐ $\cos^{-1}(\frac{1}{3})$
- ☐ $\cos^{-1}(\frac{1}{9})$
- ☑ $\cos^{-1}(\frac{\sqrt{2}}{6})$

(e) (3 points). What is the cross product $\overrightarrow{PQ} \times \overrightarrow{RS}$?

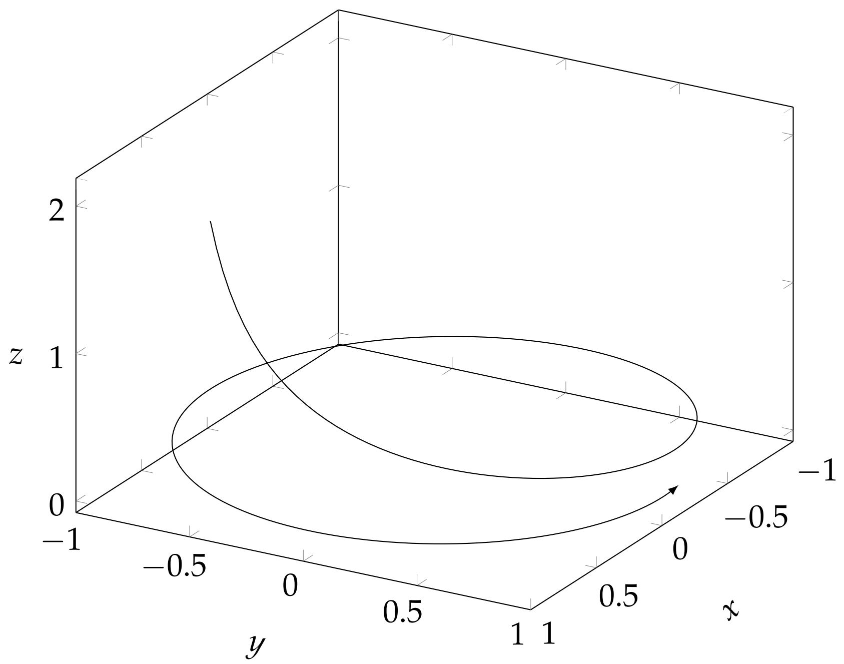
- ☑ $(-2, -3, -2)$
- ☐ $(2, 3, -1)$
- ☐ $(0, 3, 1)$
- ☐ $(-1, 3, -1)$

Problem 11 (10 points)

The position vector of a moving particle in 3D-space is given by

$$\vec{r}(t) = \big(\cos(t-1), \sin(t-1), t^{-1}\big), \quad t > \tfrac{1}{2}.$$

Here is a plot of the motion curve



(a) (2 points). At time $t = 1$ the particle is located at the point

- ☐ $(1, 1, 1)$
- ☐ $(1, -1, 2)$
- ☐ $(0, 1, 1)$
- ☑ $(1, 0, 1)$
- ☐ $(1, 1, \frac{1}{2})$
- ☐ $(0, 0, 1)$

(b) (4 points). The speed $v(t)$ of the particle is

- ☑ $\sqrt{1 + t^{-4}}$
- ☐ $(1 - t)^2$
- ☐ $\sqrt{1 + t^2}$
- ☐ $\frac{1}{1-t}$
- ☐ $2\sqrt{1 + t^4}$
- ☐ $-\sin(t-1) + \cos(t-1) - t^{-2}$

(c) (4 points). The acceleration vector at time $t = 1$ equals

- ☐ $(-1, -1, 4)$
- ☐ $(-1, 1, 2)$
- ☐ $(1, 0, 2)$
- ☑ $(-1, 0, 2)$
- ☐ $(0, 1, 1)$
- ☐ $(-1, 1, 3)$

Problem 12 (11 points)

Three matrices are given by

$$A = \begin{bmatrix} 2 & 0 & -1 & 1 \\ 0 & -3 & 1 & 0 \\ 1 & 2 & 1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 0 & 0 & 3 \\ 1 & 2 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & 1 & 0 & 2 & 7 \\ 0 & 2 & -2 & 3 & 0 \\ 1 & -3 & 2 & 1 & -1 \end{bmatrix}.$$

Mark the correct statements below.

(a) (3 points). The matrix product ABC has size

- ☐ 4×5
- ☑ 3×5
- ☐ 4×2
- ☐ 3×3
- ☐ 3×4
- ☐ 5×4

(b) (4 points). Entry $(2,3)$ of the matrix $A + 2B^T$, i.e. $[A + 2B^T]_{23}$, equals

- ☐ 3
- ☐ 0
- ☐ 12
- ☐ -1
- ☐ 4
- ☑ 1

(c) (4 points). Entry $(2,3)$ of the matrix product BC i.e. $[BC]_{23}$, equals

- ☐ 2
- ☐ -5
- ☐ 9
- ☐ 3
- ☑ 4
- ☐ 13

Problem 13 (7 points)

A matrix is given by

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix}$$

Mark the correct statement below.

- ☑ A is invertible and entry $(2,1)$ of its inverse, i.e. $[A^{-1}]_{21}$, equals -3.
- ☐ A is invertible and entry $(2,1)$ of its inverse, i.e. $[A^{-1}]_{21}$, equals 2.
- ☐ A is invertible and entry $(2,1)$ of its inverse, i.e. $[A^{-1}]_{21}$, equals -5.
- ☐ A is not invertible.
- ☐ None of the above statements apply.

Problem 14 (6 points)

A matrix and a vector are defined by

$$A = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

One can matrix multiply A by itself n times and get a new matrix A^n. What is the matrix-vector product $A^4\vec{v}$?

- ☐ $\begin{bmatrix} 81 \\ 9 \end{bmatrix}$
- ☐ $\begin{bmatrix} 20 \\ 4 \end{bmatrix}$
- ☐ $\begin{bmatrix} 10 \\ 11 \end{bmatrix}$
- ☐ $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$
- ☑ $\begin{bmatrix} 16 \\ 16 \end{bmatrix}$
- ☐ $\begin{bmatrix} 27 \\ 8 \end{bmatrix}$