Reexam in Mathematics for Multimedia Applications

First Year at The Faculty of Engineering and Science

16 August 2016

This exam set consists of 9 pages with 14 problems. For each question a number of points are indicated. The total number of points equals 100.

It is allowed to use books, notes, photocopies etc. It is **not allowed** to use **any electronic devices** such as pocket calculators, mobile phones or computers.

The exam set has two independent parts.

- Part I contains "essay problems". Here it is important that you explain the idea behind the solution, and that you provide relevant intermediate results.
- Part II contains "multiple choice" problems. **The answers of Part II must be given on these sheets.**

Remember to write your full name (including middle names) together with your student number below. Also write name and student number on each page of your solutions of the essay problems and number these pages. Indicate the total number of extra sheets on the first page.

Good luck!

NAME: ____

STUDENT NUMBER: ____

Part I (Essay-problems)

Problem 1 (9 points)

(a) (4 points). Prove that the following identity holds:

$$(\cos(x) + \sin(x))^2 + (\cos(x) - \sin(x))^2 = 2.$$

(b) (2 points). If $\cos(x) + \sin(x) = \sqrt{2}$ for an angle x what is the value of $\cos(x) - \sin(x)$?

(c) (3 points). Solve the following equation where x is the unknown:

$$\cos(x) + \sin(x) = \sqrt{2}.$$

Problem 2 (11 points)

A system of linear equations is given by

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 1 \\ -x_1 + x_2 + 4x_3 &= 1 \\ x_1 + 2x_2 + 5x_3 &= 2. \end{aligned}$$

(a) (2 points). Find the augmented matrix of the system.

(b) (5 points). Find the reduced row echelon form of the augmented matrix.

(c) (4 points). Write down the general solution of the system.

Part II (Multiple-choice problems)

Problem 3 (4 points)

A function is given by

$$f(x) = x^3 + \ln(x^4 + 1).$$

Mark the correct expression for its derivative $f'(x)$.

- ☐ $3x^2 + \frac{x^4+1}{x}$
- ☐ $\frac{1}{4}x^4 + \frac{1}{x}$
- ☐ $3x^2 + 4x^2$
- ☐ $3x^2 + \frac{4x^3}{x^4+1}$
- ☐ $\frac{1}{4}x^4 + \frac{1}{x^4+1}$
- ☐ $x^2 + \frac{1}{x^4+1}$

Problem 4 (4 points)

The graph of the function $g(x) = e^x - x$ has a horizontal tangent at a single point. What are the xy-coordinates of that point?

- ☐ $(0, 1)$
- ☐ $(0, e)$
- ☐ $(1, 1)$
- ☐ $(1, e - 1)$
- ☐ $(-1, 0)$
- ☐ $(-1, \frac{1}{e} - 1)$

Problem 5 (5 points)

What is the value of the limit

$$\lim_{h \to 0} \frac{e^{2(a+h)} - e^{2a}}{h}$$

where a is a constant?

- ☐ a^2
- ☐ e^{2a}
- ☐ 1
- ☐ ∞
- ☐ $2e^{2a}$
- ☐ $\frac{e^a}{2}$

Problem 6 (3 points)

The sum

$$\sum_{i=1}^{4}(5i - i^2)$$

is equal to

- ☐ 11
- ☐ 25
- ☐ 10
- ☐ 18
- ☐ 25
- ☐ 20

Problem 7 (5 points)

The sum

$$\sum_{i=1}^{10}(i^3 - 5i)$$

is equal to

- ☐ 11010
- ☐ 2750
- ☐ 1030
- ☐ 2400
- ☐ 3104
- ☐ 3125

Problem 8 (5 points)

A particle is moving along a horizontal axis. Its position as a function of time t is given by

$$x(t) = 3t^2 - 12t + 5.$$

(a) (2 points). At which time is the velocity of the particle equal to zero?

- ☐ 1
- ☐ -1
- ☐ 0
- ☐ 5
- ☐ 3
- ☐ 2

(b) (3 points). What is the acceleration $a(t)$ of the particle?

- ☐ $\frac{2}{3}$
- ☐ $6t - 1$
- ☐ 3
- ☐ 6
- ☐ -6
- ☐ $12t$

Problem 9 (8 points)

Evaluate the integrals below and mark the correct result.

(a) (4 points). The integral

$$\int_0^1 (e^{3x} + x^2)dx$$

is equal to

- ☐ $e^2 + 1$
- ☐ $\frac{e+1}{3}$
- ☐ e^3
- ☐ $e + \frac{1}{3}$
- ☐ $\frac{1}{3}e^3$
- ☐ 2

(b) (4 points). The integral

$$\int_0^{\frac{\pi}{2}} (3\cos(x) - \cos(3x))dx$$

is equal to

- ☐ $\frac{10}{3}$
- ☐ π
- ☐ $\frac{4}{3}$
- ☐ -2
- ☐ $\frac{\pi}{4}$
- ☐ $\frac{1}{2}$

Problem 10 (15 points)

Three points in 3D-space are given by

$$P = (1,2,-1), \quad Q = (3,1,1), \quad R = (5,2,3).$$

In consequence, we have the following two vectors:

$$\overrightarrow{PQ} = (2,-1,2), \quad \overrightarrow{PR} = (4,0,4).$$

Mark the correct answers below.

(a) (2 points). The coordinates of the vector $\overrightarrow{QR}$ are

- ☐ $(8,3,4)$
- ☐ $(1,1,0)$
- ☐ $(3,2,-1)$
- ☐ $(2,1,2)$

(b) (3 points). The line through P and Q has parametric equation

- ☐ $(x,y,z) = (3,1,1) + t(2,0,1)$
- ☐ $(x,y,z) = (3,1,1) + t(5,2,3)$
- ☐ $(x,y,z) = (1,2,-1) + t(2,-1,2)$
- ☐ $(x,y,z) = (1,2,-1) + t(3,1,1)$

(c) (3 points). The angle between the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ is

- ☐ $\cos^{-1}(\frac{2\sqrt{2}}{3})$
- ☐ $\frac{\pi}{2}$
- ☐ $\cos^{-1}(\frac{4}{3})$
- ☐ $\cos^{-1}(\frac{16}{7})$

(d) (3 points). The cross product $\overrightarrow{PQ} \times \overrightarrow{PR}$ equals

- ☐ $(-4,0,4)$
- ☐ $(3,1,2)$
- ☐ $(-4,8,-4)$
- ☐ $(-1,1,1)$

A fourth point in 3D-space is given by $S = (2,0,-1)$. A computation shows that

$$\overrightarrow{PQ} \times \overrightarrow{PS} = (4,2,-3).$$

(e) (4 points). Which one of the following points belongs to the plane through P, Q and S?

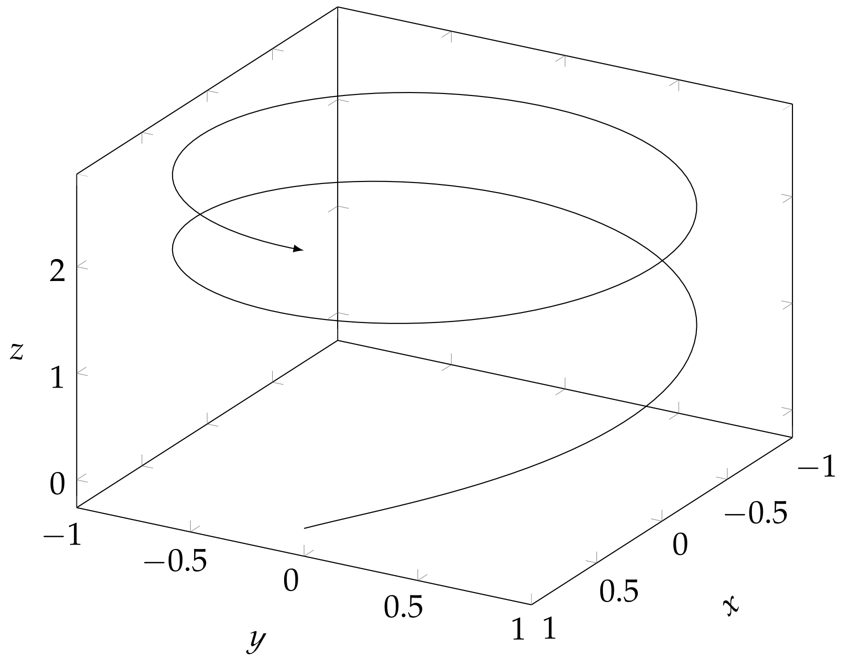
- ☐ $(1,1,1)$
- ☐ $(-1,-3,-7)$
- ☐ $(2,1,1)$
- ☐ $(-3,3,-1)$

Problem 11 (10 points)

The position vector of a moving particle in 3D-space is given by

$$\vec{r}(t) = \big(\cos(t), \sin(t), \ln(t+1)\big), \quad t > -1.$$

Here is a plot of the motion curve when the time t runs from 0 to 4π:



(a) (5 points). Mark the correct expression for the speed $v(t)$ of the particle.

- ☐ $\sqrt{2}$
- ☐ 1
- ☐ $\sqrt{\cos(t) - \sin(t) + \frac{1}{t+1}}$
- ☐ $\sqrt{1 + \frac{1}{(t+1)^2}}$
- ☐ $-\sin(t) + \cos(t) + \frac{1}{t}$
- ☐ $2\pi t$

(c) (5 points). The acceleration vector at time $t = 0$ equals

- ☐ $(-1, 0, -1)$
- ☐ $(-1, -1, 0)$
- ☐ $(0, 1, 1)$
- ☐ $(-1, 0, \sqrt{2})$
- ☐ $(0, -1, 1)$
- ☐ $(1, -1, -1)$

Problem 12 (9 points)

Three matrices are given by

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 0 \\ -1 & 1 \\ 1 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 1 & 0 & -1 \\ 1 & 0 & 2 & 5 \end{bmatrix}.$$

Mark the correct answers below.

(a) (3 points). The matrix product ABC has size

☐ 4×4	☐ 2×2	☐ 4×2
☐ 2×4	☐ 2×4	☐ 3×2

(b) (3 points). Entry $(1, 2)$ of the matrix product AB, i.e. $[AB]_{12}$, equals

☐ 0	☐ -3	☐ 4
☐ 2	☐ 1	☐ -1

(c) (3 points). Put $D = 2(A^T + B)$. Entry $(3, 2)$ of matrix D, i.e. $[D]_{32}$, equals

☐ 8	☐ 1	☐ 4
☐ -4	☐ 6	☐ 10

Problem 13 (7 points)

A matrix is given by

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Mark the correct statement below.

☐ A is invertible and entry $(2, 3)$ of its inverse, i.e. $[A^{-1}]_{23}$, equals -1.

☐ A is invertible and entry $(2, 3)$ of its inverse, i.e. $[A^{-1}]_{23}$, equals 1.

☐ A is invertible and entry $(2, 3)$ of its inverse, i.e. $[A^{-1}]_{23}$, equals 2.

☐ A is not invertible.

☐ None of the above statements apply.

Problem 14 (5 points)

Consider the 2×2 rotation matrices

$$R_\alpha = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix}, \quad R_\beta = \begin{bmatrix} \cos(\beta) & -\sin(\beta) \\ \sin(\beta) & \cos(\beta) \end{bmatrix}.$$

Which one of the following matrices equals $R_\alpha(R_\beta)^{-1}$ for any values of the rotation angles α and β?

☐ $\begin{bmatrix} \cos(\alpha)\cos(\beta) & -\sin(\alpha)\sin(\beta) \\ \sin(\alpha)\sin(\beta) & \cos(\alpha)\cos(\beta) \end{bmatrix}$ ☐ $\begin{bmatrix} \cos(\alpha-\beta) & -\sin(\alpha-\beta) \\ \sin(\alpha-\beta) & \cos(\alpha-\beta) \end{bmatrix}$

☐ $\begin{bmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix}$ ☐ $\begin{bmatrix} \cos(\alpha\beta) & -\sin(\alpha\beta) \\ \sin(\alpha\beta) & \cos(\alpha\beta) \end{bmatrix}$