

Midterm Exam in Mathematics for Multimedia Applications

First Year at the Technical Faculty of IT and Design

14 March 2018, 14:00-15:00

This exam set consists of 3 pages with 6 problems. A number of points is indicated for each question. The total number of points equals 25.

It is allowed to use books, notes, photocopies etc. It is **not allowed** to use **any electronic devices** such as pocket calculators, mobile phones or computers.

The exam set has two independent parts.

- Part I contains “essay problems”. Here it is important that you explain the idea behind the solution, and that you provide relevant intermediate results.
- Part II contains “multiple choice” problems. **The answers of Part II must be given on these sheets.**

Remember to write your full name (including middle names) together with your student number below. Also write name and student number on each page of your solutions of the essay problems and number these pages. Indicate the total number of extra sheets on the first page.

Good luck!

NAME: _____

STUDENT NUMBER: _____

Answers

Part I (Essay-problems)

Problem 1 (9 points)

- (a) (4 points). Prove that the following trigonometric identity holds:

$$\sin(x) - \sin(x) \cos^2(x) = \sin^3(x).$$

Hint: Use the fundamental identity.

- (b) (2 points). Find a solution of the equation

$$\sin(x) - \sin(x) \cos^2(x) = \frac{1}{8}.$$

Hint: $8 = 2^3$.

- (c) (3 points). Describe all solutions of the equation in (b).

(a) Insert $1 - \sin^2(x)$ in place of $\cos^2(x)$ and reduce the expression.

(b) One possible solution is $x = \frac{\pi}{6}$.

(c) $x = \frac{\pi}{6} + 2\pi p \vee x = \frac{5\pi}{6} + 2\pi p, \quad p \in \mathbb{Z}$

Part II (Multiple-choice problems)

Problem 2 (1 point)

The size of an angle is measured to 15° . What is the size of the angle measured in radians?

- $\frac{\pi}{15}$ $\frac{\pi}{8}$ $\frac{\pi}{12}$ $\frac{2\pi}{27}$ $\frac{3\pi}{14}$ 3

Problem 3 (4 points)

A particle is moving along a horizontal axis. Its position as a function of time is given by

$$x(t) = 3 \sin(2t).$$

What is the position of the particle when its velocity is 6?

- 3 3 $\frac{3\sqrt{2}}{2}$ $-\frac{3\sqrt{2}}{2}$ 0 $-\frac{1}{2}$

Problem 4 (2 points)

A function is given by

$$f(x) = (x^2 + 1)e^x.$$

What is the derivative $f'(x)$?

- x^2e^{x-1} x^2e^x $\frac{1}{3}x^3e^x$
 $2xe^{x-1}$ $2xe^x$ $(x^2 + 2x + 1)e^x$

Problem 5 (6 points)

Two functions are defined by

$$g(x) = \ln(x^2 + 2x + 3),$$
$$h(x) = e^{x^2-6x}.$$

(a) (3 points). What is the derivative $g'(x)$?

- $\frac{2x+2}{x}$ $\frac{2x}{x^2+2}$ $\frac{x^2+x}{x^2+2x+3}$
 $\frac{2x+2}{x^2+2x+3}$ $\frac{1}{x^2+2x+3}$ $\frac{1}{2x+2}$

(b) (3 points). The graph of h has a horizontal tangent at a point. What are the coordinates of that point?

- $(3, e^{-9})$ $(1, e^{-5})$ $(2, e^{-8})$
 $(0, 1)$ $(\frac{1}{2}, \frac{1}{e})$ $(-3, e^3)$

Problem 6 (3 points)

A curve is given by the equation

$$y = 2x^3 + 5x + 4.$$

What is the smallest slope that a line tangent to this curve can have?

- 2 2 4 5 7 8