Exam in Mathematics for Multimedia Applications

First Year at the Technical Faculty of IT and Design

8 November 2018, 12:30-16:30

This exam set consists of 8 pages with 13 problems. A number of points is indicated for each question. The total number of points equals 100.

It is allowed to use books, notes, photocopies etc. It is **not allowed** to use **any electronic devices** such as pocket calculators, mobile phones or computers.

The exam set has two independent parts.

- Part I contains "essay problems". Here it is important that you explain the idea behind the solution, and that you provide relevant intermediate results.
- Part II contains "multiple choice" problems. **The answers of Part II must be given on these sheets.**

Remember to write your full name (including middle names) together with your student number below. Also write name and student number on each page of your solutions of the essay problems and number these pages. Indicate the total number of extra sheets on the first page.

Good luck!

NAME: ______________________________

STUDENT NUMBER: ______________________________

Answers

Part I (Essay-problems)

Problem 1 (8 points)

(a) (5 points). Prove that the following trigonometric identity holds:

$$\sin(x)\tan(x) + \cos(x) = \frac{1}{\cos(x)}.$$

Hint: Use the fundamental identity.

(b) (3 points). Describe all solutions of the equation

$$\sin(x)\tan(x) + \cos(x) = 1.$$

(a) Multiply by $\cos(x)$ on both sides of the equality sign. Then insert

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

and use the fundamental identity.

(b) $x = 2\pi p, \quad p \in \mathbb{Z}.$

Problem 2 (13 points)

A system of linear equations is given by

$$\begin{aligned} x_1 + x_2 + x_3 - x_4 &= 0 \\ x_1 + 2x_3 &= -1 \\ 2x_1 + x_2 + 3x_3 - x_4 &= -1 \\ x_1 + 2x_2 - 2x_4 &= 1. \end{aligned}$$

(a) (3 points). Find the augmented matrix of the system.

(b) (6 points). Find the reduced row echelon form of the augmented matrix.

(c) (4 points). Write down the general solution of the system.

$$\begin{bmatrix} 1 & 1 & 1 & -1 & 0 \\ 1 & 0 & 2 & 0 & -1 \\ 2 & 1 & 3 & -1 & -1 \\ 1 & 2 & 0 & -2 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 2 & 0 & -1 \\ 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \begin{aligned} x_1 &= -1 - 2x_3 \\ x_2 &= 1 + x_3 + x_4 \\ x_3 &\text{ free} \\ x_4 &\text{ free} \end{aligned}$$

Part II (Multiple-choice problems)

Problem 3 (1 point)

The size of an angle is $\frac{\pi}{45}$ radians. What is the size of the angle measured in degrees?

- [ ] 3°
- [ ] 1°
- [x] 4°
- [ ] 14°
- [ ] 5°
- [ ] 10°
- [ ] 15°
- [ ] 12°

Problem 4 (5 points)

A particle is moving along a horizontal axis. Its position as a function of time t is given by

$$x(t) = t^3 - 3t + 4.$$

The velocity of the particle is zero at some time instance $t > 0$. What is the position of the particle when this happens?

- [ ] 3
- [ ] 0
- [x] 2
- [ ] 10
- [ ] 4
- [ ] 1
- [ ] 5
- [ ] 12

Problem 5 (5 points)

A function is given by

$$f(x) = \ln(2 + \sin(x)).$$

Mark the correct expression for its derivative $f'(x)$.

- [x] $\frac{\cos(x)}{2+\sin(x)}$
- [ ] $\frac{1}{\cos(x)}$
- [ ] $\frac{\cos(x)}{x}$
- [ ] $\frac{1}{2+\cos(x)}$
- [ ] $\frac{1}{2+\sin(x)}$
- [ ] $(2 + \cos(x)) \ln(2 + \sin(x))$
- [ ] $\frac{2+\sin(x)}{x}$
- [ ] $\frac{1-\cos(x)}{x}$

Problem 6 (10 points)

Evaluate the integrals below and mark the correct results.

(a) (5 points). The integral

$$\int_1^2 (x^{-1} + x^{-2})dx$$

is equal to

☐ $\frac{1}{3}$	☐ $\frac{6}{5}$	☐ $\frac{1}{3}\ln(3)$	☐ $\frac{\pi}{3}$
☑ $\frac{1}{2} + \ln(2)$	☐ $2\ln(2) - 1$	☐ 3	☐ $\frac{1}{7}e^2$

(b) (5 points). The integral

$$\int_0^1 \sin(\pi x)dx$$

is equal to

☐ π	☐ 1	☐ 2	☐ $\frac{\pi}{3}$
☐ $\frac{1}{4}$	☐ $\frac{1}{2}$	☐ 4π	☑ $\frac{2}{\pi}$

Problem 7 (4 points)

Mark the value of the sum

$$\sum_{i=1}^{6} (i-2)(i-3).$$

☐ 52	☑ 22	☐ 54	☐ 102
☐ 35	☐ 27	☐ 28	☐ 25

Problem 8 (8 points)

Mark the value of the sum

$$\sum_{i=1}^{20} (i-1)(i-2).$$

☐ 2010	☐ 9450	☑ 2280	☐ 2560
☐ 1130	☐ 1150	☐ 3100	☐ 1679

Problem 9 (18 points)

Three points in 3D-space are given by

$$P = (3, 1, -1), \quad Q = (5, 2, -3), \quad R = (3, 2, -2).$$

In consequence, we have the following two vectors

$$\overrightarrow{PQ} = (2, 1, -2), \quad \overrightarrow{PR} = (0, 1, -1).$$

Mark the correct answers below.

(a) (2 points). The coordinates of the vector $\overrightarrow{QR}$ are

☐ $(0, 1, -1)$ ☑ $(-2, 0, 1)$ ☐ $(1, 4, -3)$

☐ $(1, 1, -1)$ ☐ $(-1, 2, 3)$ ☐ $(-1, 2, 0)$

(b) (3 points). The line through P and Q has parametric equation

☐ $(x, y, z) = (3, 1, -1) + t(5, 2, -3)$ ☑ $(x, y, z) = (3, 1, -1) + t(2, 1, -2)$

☐ $(x, y, z) = (5, 2, -3) + t(0, 1, -1)$ ☐ $(x, y, z) = (5, 2, -3) + t(1, -1, 1)$

(c) (3 points). The cross product $\overrightarrow{PQ} \times \overrightarrow{PR}$ equals

☑ $(1, 2, 2)$ ☐ $(2, 1, -4)$ ☐ $(1, -3, -1)$

☐ $(1, 3, -4)$ ☐ $(1, 1, -1)$ ☐ $(-1, 5, 2)$

(d) (3 points). The plane through P, Q and R has equation

☐ $x - 2y - z = 1$ ☐ $3x + y - z = 5$ ☐ $x + 4y - z = 2$

☑ $x + 2y + 2z = 3$ ☐ $-x - 5 + 2z = 0$ ☐ $x + y - z = 3$

(e) (3 points). The dot product $\overrightarrow{PQ} \bullet \overrightarrow{PR}$ equals

☐ 2 ☐ 1 ☐ -5

☐ -2 ☐ 4 ☑ 3

(f) (4 points). What is the angle between the plane through P, Q and R and the plane with equation $-x + y + z = 2$?

☐ $\frac{\pi}{4}$ ☐ $\frac{\pi}{2}$ ☐ $\cos^{-1}(\frac{1}{8})$

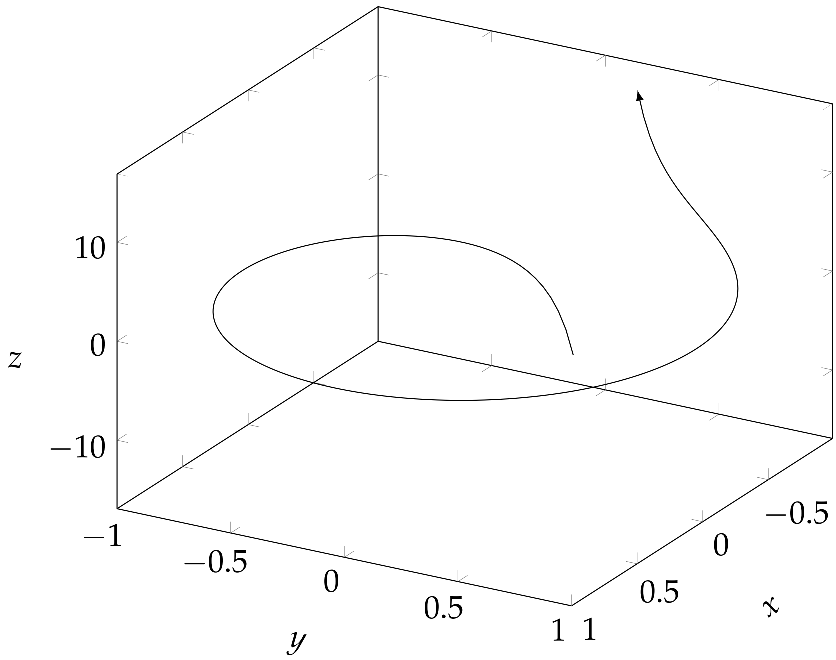
☐ $\cos^{-1}(\frac{\sqrt{5}}{2})$ ☑ $\cos^{-1}(\frac{\sqrt{3}}{3})$ ☐ $\frac{\pi}{6}$

Problem 10 (10 points)

The position vector of a moving particle in 3D-space is given by

$$\vec{r}(t) = \big(\cos(2t), \sin(2t), \tan(t)\big), \quad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

Here is a plot of the motion curve:



(a) (2 points). At time $t = 0$ the particle is located at the point

- ☐ $(1, 1, 1)$
- ☐ $(1, 0, \pi)$
- ☐ $(0, 2, \frac{\pi}{4})$
- ☑ $(1, 0, 0)$
- ☐ $(2, 0, 2)$
- ☐ $(0, 1, 1)$

(b) (4 points). The velocity vector at time $t = 0$ equals

- ☐ $(0, 2, 0)$
- ☐ $(-1, 1, \pi)$
- ☐ $(-2, 1, 1)$
- ☐ $(0, 1, \frac{\pi}{4})$
- ☑ $(0, 2, 1)$
- ☐ $(2, 2, 0)$

(c) (4 points). The speed of the particle $\nu(t)$ is

- ☐ $\sqrt{2 + \tan^2(t)}$
- ☑ $\sqrt{4 + \frac{1}{\cos^4(t)}}$
- ☐ $\sqrt{1 + \pi t}$
- ☐ $\cos(t) + \sin(t) + \cos^2(t)$
- ☐ $4 + \tan(t)$
- ☐ $(1 + \tan(t))^2$

Problem 11 (5 points)

Two matrices are given by

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 & 1 \\ 1 & 0 & 2 & 2 & 0 \\ -1 & 1 & 3 & 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 \\ 3 & 1 \\ 5 & -2 \\ 1 & 5 \\ -1 & 8 \end{bmatrix}.$$

Mark the correct statements below.

(a) (2 points). The matrix product AB has size

- ☑ 3×2
- ☐ 3×5
- ☐ 4×2
- ☐ 2×3
- ☐ 5×3
- ☐ 5×2

(b) (3 points). Entry $(2, 1)$ of the matrix product AB, i.e. $[AB]_{21}$, equals

- ☐ 12
- ☐ -3
- ☑ 13
- ☐ 7
- ☐ -5
- ☐ 8

Problem 12 (7 points)

A matrix is given by

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

Mark the correct statement below.

- ☑ A is invertible and entry $(3, 1)$ of its inverse, i.e. $[A^{-1}]_{31}$, equals 6.
- ☐ A is invertible and entry $(3, 1)$ of its inverse, i.e. $[A^{-1}]_{31}$, equals -3.
- ☐ A is invertible and entry $(3, 1)$ of its inverse, i.e. $[A^{-1}]_{31}$, equals 5.
- ☐ A is not invertible.
- ☐ None of the above statements apply.

Problem 13 (6 points)

A matrix is defined as

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

One can matrix multiply A by itself n times and get a new matrix A^n. What is the matrix A^{2018}?

☐ $\begin{bmatrix} 2018 & 0 \\ 0 & 2018 \end{bmatrix}$

☑ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

☐ $\begin{bmatrix} 1 & 0 \\ 1 & 2018 \end{bmatrix}$

☐ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

☐ $\begin{bmatrix} 1 & 2018 \\ 0 & 1 \end{bmatrix}$

☐ $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$