Exam in Mathematics for Multimedia Applications

First Year at the Technical Faculty of IT and Design

4 June 2018, 9:00-13:00

This exam set consists of 6 pages with 9 problems. A number of points is indicated for each question. The total number of points equals 75.

It is allowed to use books, notes, photocopies etc. It is **not allowed** to use **any electronic devices** such as pocket calculators, mobile phones or computers.

The exam set has two independent parts.

- Part I contains an "essay problem". Here it is important that you explain the idea behind the solution, and that you provide relevant intermediate results.
- Part II contains "multiple choice" problems. **The answers of Part II must be given on these sheets.**

Remember to write your full name (including middle names) together with your student number below. Also write name and student number on each page of your solution of the essay problem and number these pages. Indicate the total number of extra sheets on the first page.

Good luck!

NAME: ________________________________

STUDENT NUMBER: ________________________________

Part I (Essay-problem)

Problem 1 (12 points)

A system of linear equations is given by

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 4 \\ 2x_1 + 3x_2 + x_3 + 5x_4 &= 9 \\ 4x_1 + 4x_2 + 5x_3 + 6x_4 &= 18. \end{aligned}$$

(a) (3 points). Find the augmented matrix of the system.

(b) (5 points). Find the reduced row echelon form of the augmented matrix.

(c) (4 points). Write down the general solution of the system.

Part II (Multiple-choice problems)

Problem 2 (3 points)

Mark the value of the sum

$$\sum_{i=1}^{5}(i+2)(i-1).$$

☐ 48	☐ 60	☐ 21	☐ 125
☐ 110	☐ 56	☐ 120	☐ 72

Problem 3 (6 points)

Mark the value of the sum

$$\sum_{i=1}^{100}(6i-1).$$

☐ 22100	☐ 28000	☐ 21300	☐ 11500
☐ 55900	☐ 25200	☐ 30200	☐ 42200

Problem 4 (8 points)

Evaluate the integrals below and mark the correct results.

(a) (4 points). The integral

$$\int_1^3 (x^2 + x^{-2})dx$$

is equal to

□ 9	□ π^3	□ $\frac{5}{2}$	□ $\frac{1}{2}$
□ $\frac{25}{3}$	□ $\frac{\pi^3}{3}$	□ 10	□ $\frac{28}{3}$

(b) (4 points). The integral

$$\int_0^{\pi/10} \cos(5x)dx$$

is equal to

□ 2	□ π	□ $\frac{1}{5}$	□ $\frac{1}{3}$
□ 0	□ $\frac{\pi}{2}$	□ -3	□ $\frac{5}{4}$

Problem 5 (9 points)

Two matrices are given by

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 7 & 2 & -1 & 8 \\ 2 & 0 & 1 & 0 & 3 \end{bmatrix}.$$

Mark the correct statements below.

(a) (2 points). The matrix product AB has size

□ 5×2	□ 2×5	□ 4×5	□ 5×4

(b) (3 points). Entry $(1,3)$ of the matrix product AB, i.e. $[AB]_{13}$, equals

□ 2	□ -1	□ 3	□ 8
□ 5	□ 1	□ 7	□ 11

(c) (4 points). Entry $(2,2)$ of the matrix A^3 i.e. $[A^3]_{22}$, equals

□ 1	□ 8	□ 5	□ 7
□ -2	□ -8	□ 0	□ 3

Problem 6 (18 points)

Three points in 3D-space are given by

$$P = (1,2,2), \quad Q = (4,2,-2), \quad R = (3,3,4).$$

In consequence, we have the following vectors

$$\overrightarrow{PQ} = (3,0,-4), \quad \overrightarrow{PR} = (2,1,2).$$

Mark the correct answers below.

(a) (3 points). The coordinates of the vector $\overrightarrow{QR}$ are

☐ $(1,2,-1)$ ☐ $(-1,1,6)$ ☐ $(1,0,-3)$

☐ $(1,1,8)$ ☐ $(2,2,5)$ ☐ $(-1,6,2)$

(b) (3 points). The angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ is

☐ $\frac{\pi}{6}$ ☐ $\cos^{-1}(\frac{4}{5})$ ☐ $\cos^{-1}(-\frac{2}{15})$

☐ $\frac{\pi}{2}$ ☐ $\cos^{-1}(-\frac{1}{3})$ ☐ $\cos^{-1}(-\frac{8}{3})$

(c) (3 points). The cross product $\overrightarrow{PQ} \times \overrightarrow{PR}$ equals

☐ $(4,10,3)$ ☐ $(1,4,3)$ ☐ $(2,0,-1)$

☐ $(4,-14,3)$ ☐ $(-1,5,8)$ ☐ $(4,-10,2)$

(d) (3 points). The line through P and R has parametric equation

☐ $(x,y,z) = (3,3,4) + t(1,2,2)$ ☐ $(x,y,z) = (3,3,4) + t(3,3,4)$

☐ $(x,y,z) = (1,2,2) + t(2,1,2)$ ☐ $(x,y,z) = (2,1,2) + t(3,0,-4)$

☐ $(x,y,z) = (1,2,2) + t(3,0,-4)$ ☐ $(x,y,z) = (1,0,-1) + t(1,1,3)$

(e) (6 points). The line with parametric equation $(x,y,z) = (1,1,1) + t(1,1,3)$ intersect the plane through P, Q and R. What are the coordinates of the point of intersection?

☐ $(2,2,4)$ ☐ $(12,12,34)$ ☐ $(0,0,-2)$

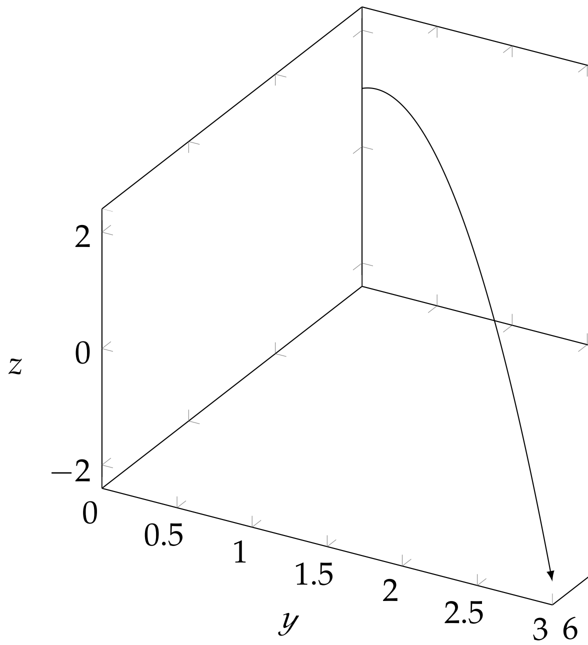
☐ $(4,1,2)$ ☐ $(4,4,10)$ ☐ $(11,11,31)$

Problem 7 (9 points)

The position vector of a moving particle in 3D-space is given by

$$\vec{r}(t) = \left(2t, t, -t^2 + 2t + 1\right).$$

Here is a plot of the motion curve when the time t runs from 0 to 3:



(a) (2 points). What are the coordinates of the particle at time $t = 2$?

- ☐ $(2, 2, 3)$
- ☐ $(5, 1, 1)$
- ☐ $(0, 0, 1)$
- ☐ $(1, 1, 3)$
- ☐ $(4, 2, 1)$
- ☐ $(0, 0, 2)$

(b) (3 points). What is the acceleration vector $\vec{a}(t)$ for the moving particle?

- ☐ $(0, 0, -2)$
- ☐ $(1, 1, 2t + 2)$
- ☐ $(2, 1, -2t)$
- ☐ $(0, 0, -2t + 2)$
- ☐ $(0, 1, -2t)$
- ☐ $(t, t, -2t + t)$

(c) (4 points). What is the speed $v(t)$ of the particle?

- ☐ $5 - 2t$
- ☐ $t^2 + 5t + 1$
- ☐ $\sqrt{4t^2 - 8t + 9}$
- ☐ $9 - 3t$
- ☐ $\sqrt{2 - 5t^2}$
- ☐ $\sqrt{5 - 2t}$

Problem 8 (6 points)

A matrix is given by

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 5 & 6 \\ -2 & -7 & -9 \end{bmatrix}.$$

Mark the correct statement below.

☐ A is invertible and entry $(2,3)$ of its inverse, i.e. $[A^{-1}]_{23}$, equals 3.

☐ A is invertible and entry $(2,3)$ of its inverse, i.e. $[A^{-1}]_{23}$, equals -1.

☐ A is invertible and entry $(2,3)$ of its inverse, i.e. $[A^{-1}]_{23}$, equals -2.

☐ A is invertible and entry $(2,3)$ of its inverse, i.e. $[A^{-1}]_{23}$, equals 0.

☐ A is not invertible.

☐ None of the above statements apply.

Problem 9 (4 points)

Two matrices are defined as

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}.$$

Both matrices are invertible and

$$A^{-1} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}, \quad B^{-1} = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}.$$

Compute the matrix $(AB)^{-1}$ and mark it below.

☐ $\begin{bmatrix} -2 & 1 \\ 1 & 5 \end{bmatrix}$ ☐ $\begin{bmatrix} 22 & -3 \\ 10 & 5 \end{bmatrix}$ ☐ $\begin{bmatrix} -8 & 3 \\ 10 & 4 \end{bmatrix}$

☐ $\begin{bmatrix} -15 & 10 \\ 10 & -5 \end{bmatrix}$ ☐ $\begin{bmatrix} -20 & 7 \\ 10 & -4 \end{bmatrix}$ ☐ $\begin{bmatrix} -18 & 7 \\ 13 & -5 \end{bmatrix}$