

# Exam in Mathematics for Multimedia Applications

First Year at the Technical Faculty of IT and Design

4 June 2018, 9:00-13:00

This exam set consists of 6 pages with 9 problems. A number of points is indicated for each question. The total number of points equals 75.

It is allowed to use books, notes, photocopies etc. It is **not allowed** to use **any electronic devices** such as pocket calculators, mobile phones or computers.

The exam set has two independent parts.

- Part I contains an “essay problem”. Here it is important that you explain the idea behind the solution, and that you provide relevant intermediate results.
- Part II contains “multiple choice” problems. **The answers of Part II must be given on these sheets.**

Remember to write your full name (including middle names) together with your student number below. Also write name and student number on each page of your solution of the essay problem and number these pages. Indicate the total number of extra sheets on the first page.

Good luck!

NAME: \_\_\_\_\_

STUDENT NUMBER: \_\_\_\_\_

## Answers

## Part I (Essay-problem)

### Problem 1 (12 points)

A system of linear equations is given by

$$\begin{aligned}x_1 + x_2 + x_3 + x_4 &= 4 \\2x_1 + 3x_2 + x_3 + 5x_4 &= 9 \\4x_1 + 4x_2 + 5x_3 + 6x_4 &= 18.\end{aligned}$$

- (a) (3 points). Find the augmented matrix of the system.  
(b) (5 points). Find the reduced row echelon form of the augmented matrix.  
(c) (4 points). Write down the general solution of the system.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 4 \\ 2 & 3 & 1 & 5 & 9 \\ 4 & 4 & 5 & 6 & 18 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 0 & -6 & -1 \\ 0 & 1 & 0 & 5 & 3 \\ 0 & 0 & 1 & 2 & 2 \end{bmatrix}, \quad \begin{aligned}x_1 &= -1 + 6x_4 \\x_2 &= 3 - 5x_4 \\x_3 &= 2 - 2x_4 \\x_4 &\text{ free}\end{aligned}$$

## Part II (Multiple-choice problems)

### Problem 2 (3 points)

Mark the value of the sum

$$\sum_{i=1}^5 (i+2)(i-1).$$

- 48                       60                       21                       125  
 110                       56                       120                       72

### Problem 3 (6 points)

Mark the value of the sum

$$\sum_{i=1}^{100} (6i-1).$$

- 22100                       28000                       21300                       11500  
 55900                       25200                       30200                       42200

### Problem 4 (8 points)

Evaluate the integrals below and mark the correct results.

(a) (4 points). The integral

$$\int_1^3 (x^2 + x^{-2}) dx$$

is equal to

- |                                         |                                            |                                        |                                                    |
|-----------------------------------------|--------------------------------------------|----------------------------------------|----------------------------------------------------|
| <input type="checkbox"/> 9              | <input type="checkbox"/> $\pi^3$           | <input type="checkbox"/> $\frac{5}{2}$ | <input type="checkbox"/> $\frac{1}{2}$             |
| <input type="checkbox"/> $\frac{25}{3}$ | <input type="checkbox"/> $\frac{\pi^3}{3}$ | <input type="checkbox"/> 10            | <input checked="" type="checkbox"/> $\frac{28}{3}$ |

(b) (4 points). The integral

$$\int_0^{\pi/10} \cos(5x) dx$$

is equal to

- |                            |                                          |                                                   |                                        |
|----------------------------|------------------------------------------|---------------------------------------------------|----------------------------------------|
| <input type="checkbox"/> 2 | <input type="checkbox"/> $\pi$           | <input checked="" type="checkbox"/> $\frac{1}{5}$ | <input type="checkbox"/> $\frac{1}{3}$ |
| <input type="checkbox"/> 0 | <input type="checkbox"/> $\frac{\pi}{2}$ | <input type="checkbox"/> -3                       | <input type="checkbox"/> $\frac{5}{4}$ |

### Problem 5 (9 points)

Two matrices are given by

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 7 & 2 & -1 & 8 \\ 2 & 0 & 1 & 0 & 3 \end{bmatrix}.$$

Mark the correct statements below.

(a) (2 points). The matrix product  $AB$  has size

- |                                       |                                                  |                                       |                                       |
|---------------------------------------|--------------------------------------------------|---------------------------------------|---------------------------------------|
| <input type="checkbox"/> $5 \times 2$ | <input checked="" type="checkbox"/> $2 \times 5$ | <input type="checkbox"/> $4 \times 5$ | <input type="checkbox"/> $5 \times 4$ |
|---------------------------------------|--------------------------------------------------|---------------------------------------|---------------------------------------|

(b) (3 points). Entry  $(1,3)$  of the matrix product  $AB$ , i.e.  $[AB]_{13}$ , equals

- |                            |                                       |                            |                             |
|----------------------------|---------------------------------------|----------------------------|-----------------------------|
| <input type="checkbox"/> 2 | <input type="checkbox"/> -1           | <input type="checkbox"/> 3 | <input type="checkbox"/> 8  |
| <input type="checkbox"/> 5 | <input checked="" type="checkbox"/> 1 | <input type="checkbox"/> 7 | <input type="checkbox"/> 11 |

(c) (4 points). Entry  $(2,2)$  of the matrix  $A^3$  i.e.  $[A^3]_{22}$ , equals

- |                                        |                             |                            |                            |
|----------------------------------------|-----------------------------|----------------------------|----------------------------|
| <input type="checkbox"/> 1             | <input type="checkbox"/> 8  | <input type="checkbox"/> 5 | <input type="checkbox"/> 7 |
| <input checked="" type="checkbox"/> -2 | <input type="checkbox"/> -8 | <input type="checkbox"/> 0 | <input type="checkbox"/> 3 |

### Problem 6 (18 points)

Three points in 3D-space are given by

$$P = (1, 2, 2), \quad Q = (4, 2, -2), \quad R = (3, 3, 4).$$

In consequence, we have the following vectors

$$\overrightarrow{PQ} = (3, 0, -4), \quad \overrightarrow{PR} = (2, 1, 2).$$

Mark the correct answers below.

(a) (3 points). The coordinates of the vector  $\overrightarrow{QR}$  are

- |                                     |                                                |                                     |
|-------------------------------------|------------------------------------------------|-------------------------------------|
| <input type="checkbox"/> (1, 2, -1) | <input checked="" type="checkbox"/> (-1, 1, 6) | <input type="checkbox"/> (1, 0, -3) |
| <input type="checkbox"/> (1, 1, 8)  | <input type="checkbox"/> (2, 2, 5)             | <input type="checkbox"/> (-1, 6, 2) |

(b) (3 points). The angle between  $\overrightarrow{PQ}$  and  $\overrightarrow{PR}$  is

- |                                          |                                                    |                                                                |
|------------------------------------------|----------------------------------------------------|----------------------------------------------------------------|
| <input type="checkbox"/> $\frac{\pi}{6}$ | <input type="checkbox"/> $\cos^{-1}(\frac{4}{5})$  | <input checked="" type="checkbox"/> $\cos^{-1}(-\frac{2}{15})$ |
| <input type="checkbox"/> $\frac{\pi}{2}$ | <input type="checkbox"/> $\cos^{-1}(-\frac{1}{3})$ | <input type="checkbox"/> $\cos^{-1}(-\frac{8}{3})$             |

(c) (3 points). The cross product  $\overrightarrow{PQ} \times \overrightarrow{PR}$  equals

- |                                                 |                                     |                                      |
|-------------------------------------------------|-------------------------------------|--------------------------------------|
| <input type="checkbox"/> (4, 10, 3)             | <input type="checkbox"/> (1, 4, 3)  | <input type="checkbox"/> (2, 0, -1)  |
| <input checked="" type="checkbox"/> (4, -14, 3) | <input type="checkbox"/> (-1, 5, 8) | <input type="checkbox"/> (4, -10, 2) |

(d) (3 points). The line through  $P$  and  $R$  has parametric equation

- |                                                                          |                                                                |
|--------------------------------------------------------------------------|----------------------------------------------------------------|
| <input type="checkbox"/> $(x, y, z) = (3, 3, 4) + t(1, 2, 2)$            | <input type="checkbox"/> $(x, y, z) = (3, 3, 4) + t(3, 3, 4)$  |
| <input checked="" type="checkbox"/> $(x, y, z) = (1, 2, 2) + t(2, 1, 2)$ | <input type="checkbox"/> $(x, y, z) = (2, 1, 2) + t(3, 0, -4)$ |
| <input type="checkbox"/> $(x, y, z) = (1, 2, 2) + t(3, 0, -4)$           | <input type="checkbox"/> $(x, y, z) = (1, 0, -1) + t(1, 1, 3)$ |

(e) (6 points). The line with parametric equation  $(x, y, z) = (1, 1, 1) + t(1, 1, 3)$  intersect the plane through  $P$ ,  $Q$  and  $R$ . What are the coordinates of the point of intersection?

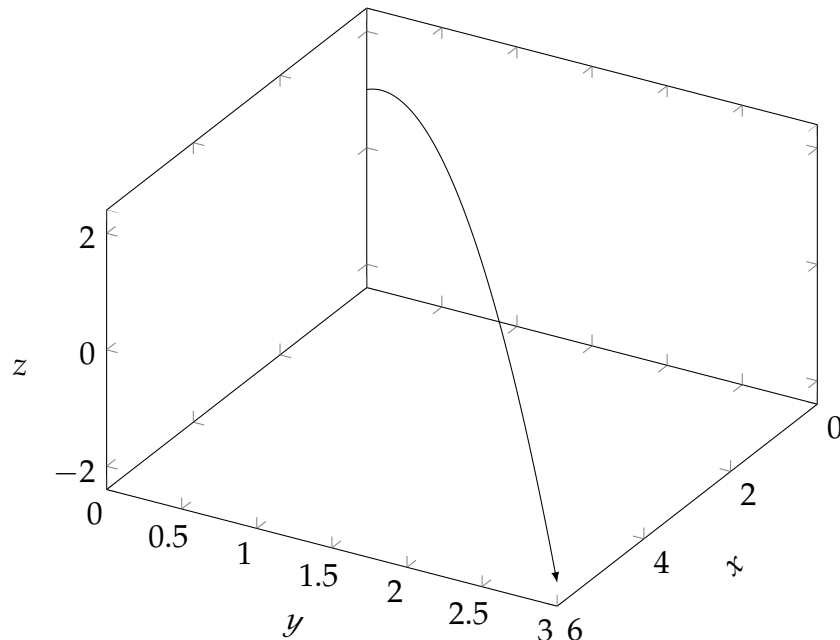
- |                                    |                                                  |                                       |
|------------------------------------|--------------------------------------------------|---------------------------------------|
| <input type="checkbox"/> (2, 2, 4) | <input checked="" type="checkbox"/> (12, 12, 34) | <input type="checkbox"/> (0, 0, -2)   |
| <input type="checkbox"/> (4, 1, 2) | <input type="checkbox"/> (4, 4, 10)              | <input type="checkbox"/> (11, 11, 31) |

### Problem 7 (9 points)

The position vector of a moving particle in 3D-space is given by

$$\vec{r}(t) = (2t, t, -t^2 + 2t + 1).$$

Here is a plot of the motion curve when the time  $t$  runs from 0 to 3:



(a) (2 points). What are the coordinates of the particle at time  $t = 2$ ?

- |                                             |                                  |
|---------------------------------------------|----------------------------------|
| <input type="checkbox"/> (2,2,3)            | <input type="checkbox"/> (5,1,1) |
| <input type="checkbox"/> (0,0,1)            | <input type="checkbox"/> (1,1,3) |
| <input checked="" type="checkbox"/> (4,2,1) | <input type="checkbox"/> (0,0,2) |

(b) (3 points). What is the acceleration vector  $\vec{a}(t)$  for the moving particle?

- |                                              |                                      |
|----------------------------------------------|--------------------------------------|
| <input checked="" type="checkbox"/> (0,0,-2) | <input type="checkbox"/> (1,1,2t+2)  |
| <input type="checkbox"/> (2,1,-2t)           | <input type="checkbox"/> (0,0,-2t+2) |
| <input type="checkbox"/> (0,1,-2t)           | <input type="checkbox"/> (t,t,-2t+t) |

(c) (4 points). What is the speed  $v(t)$  of the particle?

- |                                                            |                                          |
|------------------------------------------------------------|------------------------------------------|
| <input type="checkbox"/> $5 - 2t$                          | <input type="checkbox"/> $t^2 + 5t + 1$  |
| <input checked="" type="checkbox"/> $\sqrt{4t^2 - 8t + 9}$ | <input type="checkbox"/> $9 - 3t$        |
| <input type="checkbox"/> $\sqrt{2 - 5t^2}$                 | <input type="checkbox"/> $\sqrt{5 - 2t}$ |

### Problem 8 (6 points)

A matrix is given by

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 5 & 6 \\ -2 & -7 & -9 \end{bmatrix}.$$

Mark the correct statement below.

- $A$  is invertible and entry  $(2,3)$  of its inverse, i.e.  $[A^{-1}]_{23}$ , equals 3.
- $A$  is invertible and entry  $(2,3)$  of its inverse, i.e.  $[A^{-1}]_{23}$ , equals  $-1$ .
- $A$  is invertible and entry  $(2,3)$  of its inverse, i.e.  $[A^{-1}]_{23}$ , equals  $-2$ .
- $A$  is invertible and entry  $(2,3)$  of its inverse, i.e.  $[A^{-1}]_{23}$ , equals 0.
- $A$  is not invertible.
- None of the above statements apply.

### Problem 9 (4 points)

Two matrices are defined as

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}.$$

Both matrices are invertible and

$$A^{-1} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}, \quad B^{-1} = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}.$$

Compute the matrix  $(AB)^{-1}$  and mark it below.

- $\begin{bmatrix} -2 & 1 \\ 1 & 5 \end{bmatrix}$
- $\begin{bmatrix} 22 & -3 \\ 10 & 5 \end{bmatrix}$
- $\begin{bmatrix} -8 & 3 \\ 10 & 4 \end{bmatrix}$
- $\begin{bmatrix} -15 & 10 \\ 10 & -5 \end{bmatrix}$
- $\begin{bmatrix} -20 & 7 \\ 10 & -4 \end{bmatrix}$
- $\begin{bmatrix} -18 & 7 \\ 13 & -5 \end{bmatrix}$