Exam 2014

Mathematics for Multimedia Applications
Medialogy

3 June 2014

Formalities

This exam set consists of 4 pages, in which there are 8 problems. You are allowed to use books, notes etc. You are *not* allowed to use electronic devices such as calculators, computers or cell phones.

A number of points is indicated for every sub-problem. The sum of these points equals 100.

Date and time for the exam: 3 June, 9:00 - 13:00.

You must indicate the following on each page:

- Full name
- Study number
- Page number

On the first page, you must indicate

- The total number of pages.

Good luck!

Problems

Problem 1.

1.a. (4 points) Differentiate the function $f(x) = \sin(x^3)$.

1.b. (4 points) Differentiate the function $g(x) = (x^2 + 3x + 1)e^x$.

1.c. (3 points) The graph of the function $g(x)$ above has a tangent at the point $(0, g(0)) = (0, 1)$. What is the slope of that tangent?

Problem 2.

2.a. (4 points) Prove that the following trigonometric identity holds:

$$(\cos(x) - \sin(x))^2 = 1 - \sin(2x)$$

Hint: Use the double angle formula for sine.

2.b. (2 points) Find a solution of the equation $(\cos(x) - \sin(x))^2 = 0$.

2.c. (4 points) Describe all solutions of the equation $(\cos(x) - \sin(x))^2 = 0$.

Problem 3.

3.a. (3 points) Calculate the sum

$$\sum_{i=1}^{4} (2i^2 - i)$$

3.b. (5 points) Calculate the sum

$$\sum_{i=1}^{99} (6i^2 + 2i)$$

Problem 4. Evaluate the following integrals:

4.a. (5 points) $\int_0^{\pi/4} \sin(4x) \mathrm{d}x$

4.b. (5 points) $\int_0^1 (3x^2 + e^x) \mathrm{d}x$

Problem 5. Let P, Q and R be points in 3D-space with coordinates $(3, 5, -2)$, $(5, 3, -1)$ and $(4, 5, -1)$ respectively.

5.a. (3 points) Find $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

5.b. (2 points) Compute the dot product $\overrightarrow{PQ} \bullet \overrightarrow{PR}$.

5.c. (3 points) Find the angle between the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

5.d. (3 points) Compute the cross product $\overrightarrow{PQ} \times \overrightarrow{PR}$.

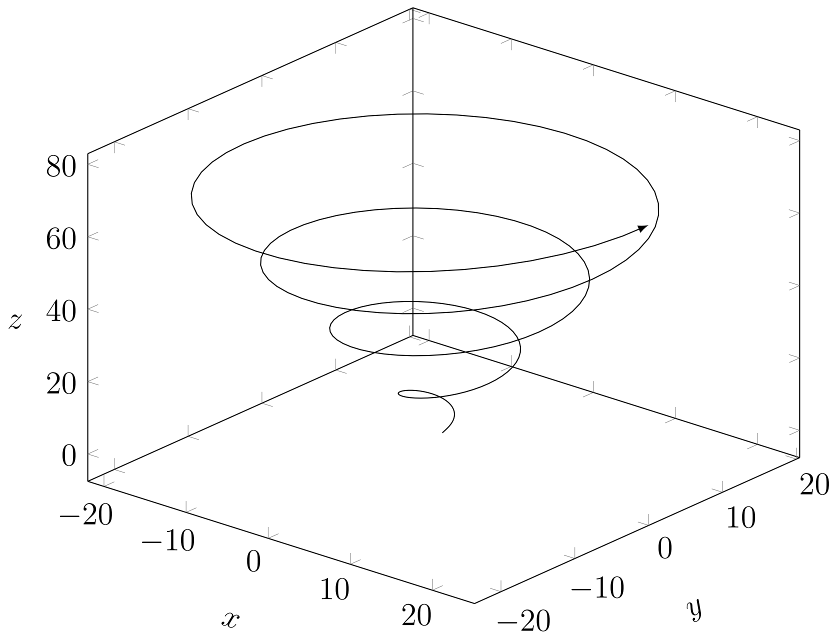
5.e. (3 points) Find the area of the triangle with vertices P, Q and R.

5.f. (3 points) Find an equation for the plane through P, Q and R.

Problem 6. The position vector of a moving particle in 3D-space is given by

$$\vec{r}(t) = \big(t \cdot \cos(t), t \cdot \sin(t), 3t\big)$$

Here is a plot of the motion curve when the time t runs from 0 to 8π:



6.a. (4 points) Compute the velocity vector $\vec{v}(t)$.

6.b. (3 points) Find the velocity vector at time $t = 0$. Compute the speed at time $t = 0$.

6.c. (1 points) Find the position of the particle at time $t = 0$.

6.d. (3 points) Find parametric equations of the tangent line to the motion curve at time $t = 0$.

Problem 7. Consider the following system of linear equations:

$$\begin{aligned} x_1 + 2x_2 &= 3 \\ -x_1 - x_2 + 7x_3 &= -1 \\ 2x_1 + 5x_2 + 7x_3 &= 8 \end{aligned}$$

7.a. (3 points) Is $x_1 = -15$, $x_2 = 9$, $x_3 = -1$ a solution of the system? Why/why not?

7.b. (3 points) Find the augmented matrix of the system.

7.c. (6 points) Find the reduced row echelon form of the augmented matrix.

7.d. (4 points) Write down the general solution of the system.

7.e. (4 points) Consider two planes in 3D-space with equations $x + 2y = 3$ and $-x - y + 7z = -1$. The planes intersect at a line. Find parametric equations of that line.

Problem 8. Define three matrices as follows:

$$A = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

8.a. (3 points) Compute $2A^T + B$.

8.b. (4 points) Compute the matrix product AB.

8.c. (6 points) Determine whether C is invertible. If so, find its inverse.

Appendix

Exact values for trigonometric functions of various angles.

	0°	30°	45°	60°	90°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0