Answers

Exam 2014

Mathematics for Multimedia Applications
Medialogy

3 June 2014

Formalities

This exam set consists of 4 pages, in which there are 8 problems. You are allowed to use books, notes etc. You are *not* allowed to use electronic devices such as calculators, computers or cell phones.

A number of points is indicated for every sub-problem. The sum of these points equals 100.

Date and time for the exam: 3 June, 9:00 - 13:00.

You must indicate the following on each page:

- Full name
- Study number
- Page number

On the first page, you must indicate

- The total number of pages.

Good luck!

Problems

Problem 1.

1.a. (4 points) Differentiate the function $f(x) = \sin(x^3)$. $f'(x) = 3x^2 \cdot \cos(x^3)$

1.b. (4 points) Differentiate the function $g(x) = (x^2 + 3x + 1)e^x$. $g'(x) = (x^2+5x+4)e^x$

1.c. (3 points) The graph of the function $g(x)$ above has a tangent at the point $(0, g(0)) = (0, 1)$. What is the slope of that tangent? $g'(0) = 4$

Problem 2.

2.a. (4 points) Prove that the following trigonometric identity holds:

$$(\cos(x) - \sin(x))^2 = 1 - \sin(2x)$$ see below

Hint: Use the double angle formula for sine.

2.b. (2 points) Find a solution of the equation $(\cos(x) - \sin(x))^2 = 0$. $x = \frac{\pi}{4}$

2.c. (4 points) Describe all solutions of the equation $(\cos(x) - \sin(x))^2 = 0$. $x = \frac{\pi}{4} + \pi p,\ p \in \mathbb{Z}$

2.a. $(\cos(x) - \sin(x))^2 = \underbrace{\cos^2(x) + \sin^2(x)}_{1} - \underbrace{2\cos(x)\sin(x)}_{\sin(2x)} = 1 - \sin(2x)$

Problem 3.

3.a. (3 points) Calculate the sum

$$\sum_{i=1}^{4}(2i^2 - i)$$ 50

3.b. (5 points) Calculate the sum

$$\sum_{i=1}^{99}(6i^2 + 2i)$$ 1 980 000

Problem 4. Evaluate the following integrals:

4.a. (5 points) $\int_0^{\pi/4} \sin(4x)\mathrm{d}x$ $\frac{1}{2}$

4.b. (5 points) $\int_0^1 (3x^2 + e^x)\mathrm{d}x$ e

Problem 5. Let P, Q and R be points in 3D-space with coordinates $(3,5,-2)$, $(5,3,-1)$ and $(4,5,-1)$ respectively.

5.a. (3 points) Find $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. $\overrightarrow{PQ} = (2,-2,1)$, $\overrightarrow{PR} = (1,0,1)$

5.b. (2 points) Compute the dot product $\overrightarrow{PQ} \bullet \overrightarrow{PR}$. 3

5.c. (3 points) Find the angle between the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. $\frac{\pi}{4}$

5.d. (3 points) Compute the cross product $\overrightarrow{PQ} \times \overrightarrow{PR}$. $(-2,-1,2)$

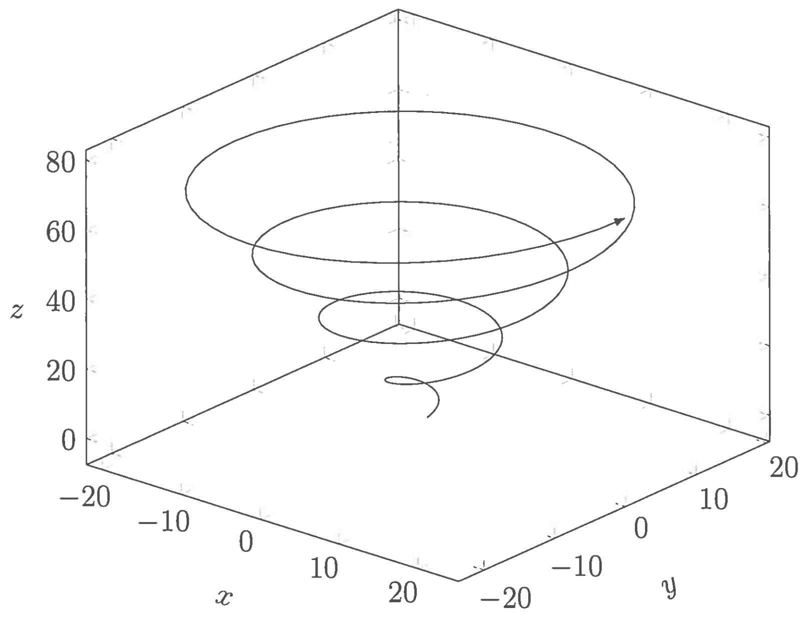
5.e. (3 points) Find the area of the triangle with vertices P, Q and R. $\frac{3}{2}$

5.f. (3 points) Find an equation for the plane through P, Q and R. $-2x - y + 2z + 15 = 0$

Problem 6. The position vector of a moving particle in 3D-space is given by

$$\vec{r}(t) = \big(t \cdot \cos(t), t \cdot \sin(t), 3t\big)$$

Here is a plot of the motion curve when the time t runs from 0 to 8π:



6.a. (4 points) Compute the velocity vector $\vec{v}(t)$. $\vec{v}(t) = (\cos(t) - t\sin(t), \sin(t) + t \cdot \cos(t), 3)$

6.b. (3 points) Find the velocity vector at time $t = 0$. Compute the speed at time $t = 0$.

$\vec{v}(0) = (1,0,3)$, $\nu(0) = \sqrt{10}$

6.c. (1 points) Find the position of the particle at time $t = 0$. $\vec{r}(0) = (0,0,0)$

6.d. (3 points) Find parametric equations of the tangent line to the motion curve at time $t = 0$.

$(x,y,z) = (0,0,0) + s(1,0,3)$ or

$x = s, \; y = 0, \; z = 3s$

Problem 7. Consider the following system of linear equations:

$$\begin{aligned} x_1 + 2x_2 &= 3 \\ -x_1 - x_2 + 7x_3 &= -1 \\ 2x_1 + 5x_2 + 7x_3 &= 8 \end{aligned}$$

7.a. (3 points) Is $x_1 = -15$, $x_2 = 9$, $x_3 = -1$ a solution of the system? Why/why not? Yes, by substituting the values in the equations we obtain the equalities.

7.b. (3 points) Find the augmented matrix of the system. $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ -1 & -1 & 7 & -1 \\ 2 & 5 & 7 & 8 \end{array}\right]$

7.c. (6 points) Find the reduced row echelon form of the augmented matrix. $\left[\begin{array}{ccc|c} 1 & 0 & -14 & -1 \\ 0 & 1 & 7 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

7.d. (4 points) Write down the general solution of the system. $\begin{cases} x_1 = -1 + 14x_3 \\ x_2 = 2 - 7x_3 \\ x_3 \text{ free} \end{cases}$

7.e. (4 points) Consider two planes in 3D-space with equations $x + 2y = 3$ and $-x - y + 7z = -1$. The planes intersect at a line. Find parametric equations of that line.

$(x, y, z) = (-1, 2, 0) + t(14, -7, 1)$ or

$x = -1 + 14t, \; y = 2 - 7t, \; z = t$

Problem 8. Define three matrices as follows:

$$A = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

8.a. (3 points) Compute $2A^T + B$. $\begin{bmatrix} 4 & 5 \\ -1 & 2 \end{bmatrix}$

8.b. (4 points) Compute the matrix product AB. $\begin{bmatrix} 1 & -3 \\ 6 & -3 \end{bmatrix}$

8.c. (6 points) Determine whether C is invertible. If so, find its inverse.

C is invertible and $C^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ -1 & -1 & 1 \end{bmatrix}$

Appendix

Exact values for trigonometric functions of various angles.

	0°	30°	45°	60°	90°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0