Exam 2013

Answers

Mathematics for Multimedia Applications
Medialogy

11. June 2013

Formalities

This exam set consists of 6 pages, in which there are 10 problems. You are allowed to use books, notes etc. You are *not* allowed to use electronic devices such as calculators, computers or cell phones.

A number of points is indicated for every sub-problem. The sum of these points equals 100.

Date and time for the exam: 11. June, 9:00 - 13:00.

You must indicate the following on each page:

- Full name
- Study number
- Page number

On the first page, you must indicate

- The total number of pages.

Good luck!

Problems

Problem 1.

1.a. (3 points) Differentiate the function $f(x) = \sin(5x) + 3\cos(x)$.

$f'(x) = 5\cos(5x) - 3\sin(x)$

1.b. (3 points) Differentiate the function $g(x) = \ln(e^x + 2)$.

$g'(x) = \dfrac{e^x}{e^x + 2}$

Problem 2.

Problem 2. The position function of a particle moving along a horizontal straight line is given by

$$x(t) = 3t^2 - 12t + 7$$

$v(t) = 6t - 12$

$a(t) = 6$

2.a. (3 points) Find the velocity $v(t)$ and the acceleration $a(t)$ of the particle.

2.b. (3 points) Find the position of the particle when its velocity is zero.

$x(2) = \underline{\underline{-5}}$

Problem 3.

3.a. (2 points) Find a solution of the equation $\sin^2(x) = 0$.

$x = 0$

3.b. (3 points) Describe all solutions of the equation $\sin^2(x) = 0$.

$x = \pi p, \ p \in \mathbb{Z}$

3.c. (3 points) Prove that the following trigonometric identity holds:

$$(1 + \cos(\theta))(1 - \cos(\theta)) = \sin^2(\theta)$$

$(1+\cos(\theta)) \cdot (1-\cos(\theta)) = 1 + \cos(\theta) - \cos(\theta) - \cos^2(\theta) = 1 - \cos^2(\theta) = \sin^2(\theta)$

since $\cos^2(\theta) + \sin^2(\theta) = 1$

Problem 4.

4.a. (3 points) Calculate the sum

$$\sum_{i=1}^{4} (i^2 - 1)$$

26

4.b. (4 points) Calculate the sum

$$\sum_{i=1}^{100} (3i^2 - i)$$

1 010 000

$F(x) = x^2 - \frac{1}{3}x^4$

Problem 5. Consider the two functions $f(x) = 2x - \frac{4}{3}x^3$ and $g(x) = \sin(2x)$. $G(x) = -\frac{1}{2}\cos(2x)$

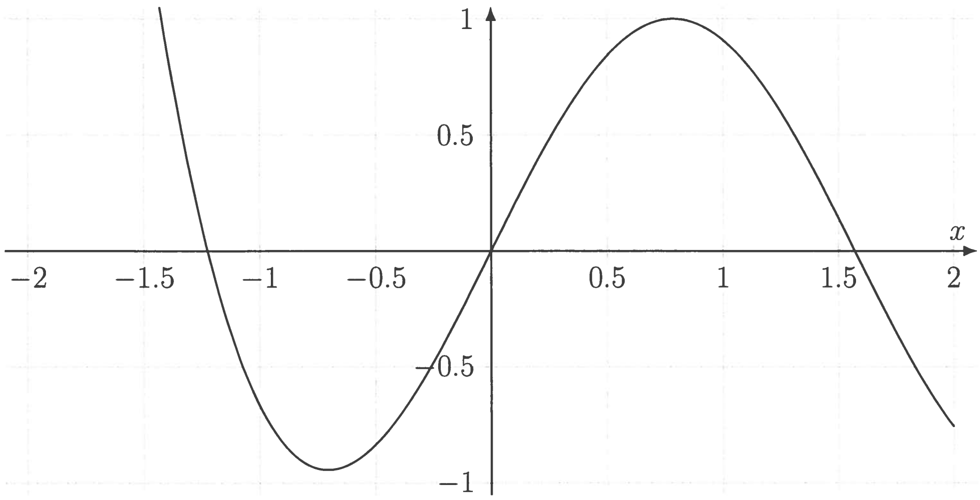
5.a. (4 points) Find antiderivatives $F(x)$ and $G(x)$ of the functions $f(x)$ and $g(x)$.

5.b. (3 points) Evaluate the integrals $\int_{-1}^{0} f(x)\mathrm{d}x$ and $\int_{0}^{\pi/2} g(x)\mathrm{d}x$. $-\frac{2}{3}$ and 1

Define the function $h(x)$ by

$$h(x) = \begin{cases} f(x) & \text{if } x < 0, \\ g(x) & \text{if } x \geq 0, \end{cases}$$

where $f(x)$ and $g(x)$ are the functions above. The graph of $h(x)$ looks as follows:



5.c. (3 points) Find $\int_{-1}^{\pi/2} h(x)\mathrm{d}x$. $\frac{1}{3}$

Problem 6. Let P, Q, R and S be points in 3D-space with coordinates $(1, 3, -1)$, $(2, 3, 0)$, $(4, 3, 2)$ and $(6, 5, 3)$ respectively.

$\overrightarrow{PQ} = (1, 0, 1)$

6.a. (2 points) Find $\overrightarrow{PQ}$ and $\overrightarrow{RS}$. $\overrightarrow{RS} = (2, 2, 1)$

6.b. (3 points) Find parametric equations for the line ℓ_1 that passes through P and Q and the line ℓ_2 that passes through R and S. $(x, y, z) = (1, 3, -1) + t(1, 0, 1)$

$(x, y, z) = (4, 3, 2) + s(2, 2, 1)$

6.c. (3 points) Show that the two lines ℓ_1 and ℓ_2 intersect at the point R.

Put $s = 0$, $t = 3$.

6.d. (2 points) Compute the dot product $\overrightarrow{PQ} \bullet \overrightarrow{RS}$. 3

6.e. (3 points) Compute the angle between the lines ℓ_1 and ℓ_2. $\frac{\pi}{4}$

Problem 7. Let O, P and Q be the points in 3D-space with coordinates $(0,0,0)$, $(1,0,1)$ and $(3,1,-1)$ respectively.

7.a. (3 points) Compute the cross product $\overrightarrow{OP} \times \overrightarrow{OQ}$. $(-1, 4, 1)$

7.b. (3 points) Find the area of the triangel with vertices O, P and Q. $\frac{\sqrt{18}}{2}$

7.c. (3 points) Find an equation for the plane through O, P and Q. $-x+4y+z=0$

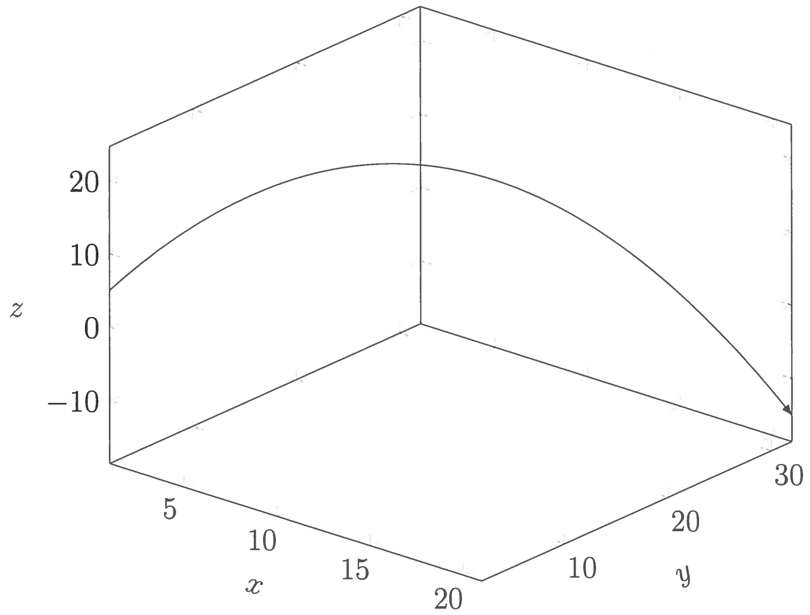
Consider the plane in 3D-space with equation $2x - y + 3z = 6$ and the line with parametric equation $(x, y, z) = (1, 0, 2) + t(-1, 3, 1)$. The plane and the line intersect at a point.

7.d. (4 points) Find the coordinates of the point of intersection. $(0, 3, 3)$

Problem 8. The position vector of a moving particle in 3D-space is given by

$$\vec{r}(t) = (3t+1, 4t+2, -t^2+8t+5).$$

Here is a plot of the motion curve when the time t runs from 0 to 10:



8.a. (3 points) Compute the velocity vector $\vec{v}(t)$. $\vec{v}(t) = (3, 4, -2t+8)$

8.b. (2 points) Compute the speed $\nu(t)$. $\nu(t) = \sqrt{25 + (-2t+8)^2}$

8.c. (1 points) Find the speed at time $t = 4$. $\nu(4) = 5$

8.d. (3 points) Find a unit vector, which points in the direction that the particle is moving at time $t = 4$.

$\left(\frac{3}{5}, \frac{4}{5}, 0\right)$

9b $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 3 & 1 & 4 \\ 1 & 2 & 0 & 3 \end{array}\right]$

9c $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

9d $\left[\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Problem 9. Consider the following system of linear equations:

$$\begin{aligned} x_1 + x_2 + x_3 &= 1 \\ 2x_1 + 3x_2 + x_3 &= 4 \\ x_1 + 2x_2 &= 3 \end{aligned}$$

9.a. (2 points) Is $x_1 = 2$, $x_2 = 1$, $x_3 = 1$ a solution of the system? Why/why not? No, insert.

9.b. (2 points) Find the augmented matrix of the system.

9.c. (4 points) Find a row echelon form of the augmented matrix.

9.d. (2 points) Find the reduced row echelon form of the augmented matrix.

9.e. (4 points) Write down the general solution of the system.

$\begin{cases} x_1 = -1 - 2x_3 \\ x_2 = 2 + x_3 \\ x_3 \text{ free} \end{cases}$

9.f. (3 points) Find a solution of the system which has $x_3 = -1$.

$\begin{cases} x_1 = 1 \\ x_2 = 1 \\ x_3 = -1 \end{cases}$

Problem 10. Define two matrices as follows:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 7 & 0 \\ 2 & 6 & 2 \end{bmatrix}$$

10.a. (2 points) Compute $2A + B$.

$\begin{bmatrix} 3 & 3 & 3 \\ 0 & 9 & 0 \\ 0 & 6 & 2 \end{bmatrix}$

10.b. (3 points) Compute the matrix product AB.

$\begin{bmatrix} 3 & 9 & 3 \\ 0 & 7 & 0 \\ -1 & -3 & -1 \end{bmatrix}$

10.c. (3 points) Determine whether A is invertible. If so, find its inverse.

10.d. (3 points) Determine whether B is invertible. If so, find its inverse.

10c A is invertible and $A^{-1} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

10d B is not invertible.

Appendix

Exact values for trigonometric functions of various angles.

	$30°$	$45°$	$60°$
	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$