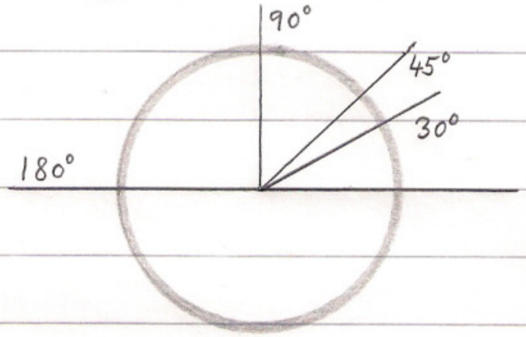
1. Session : Review of Trigonometry

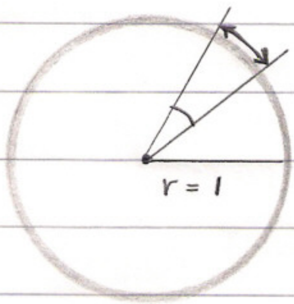
Angle measurement

- Degree measure :



- Radian measure :

Angle measurement by arc length on the unit circle.



The circumference C of the unit circle is

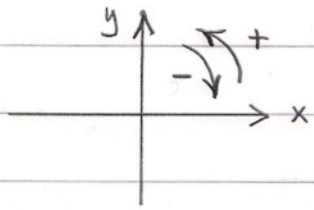
$C = 2\pi r = 2\pi \cdot 1 = 2\pi$

rad.	2π	π	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$
deg.	360°	180°	90°	60°	45°	30°

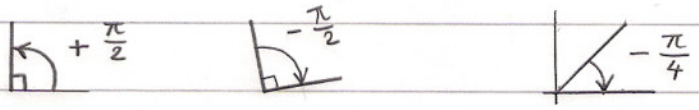
Since π rad $= 180°$, we have

$\underline{1 \text{ rad} = \frac{180°}{\pi}}$ $\qquad$ $\underline{1° = \frac{\pi}{180} \text{ rad}}$

Positive / negative angles :

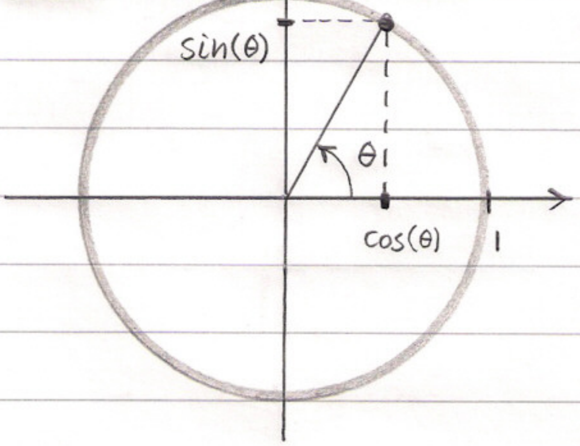


Ex.



Trigonometric functions

!Def.



$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$
$$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

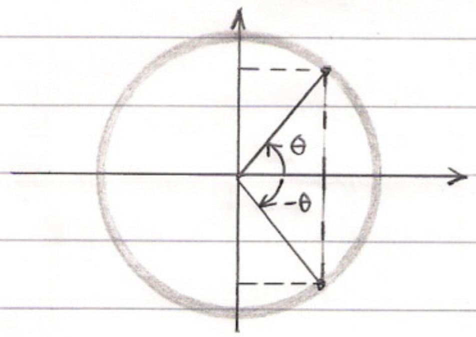
$$\sec(\theta) = \frac{1}{\cos(\theta)}$$
$$\csc(\theta) = \frac{1}{\sin(\theta)}$$

Trigonometric identities

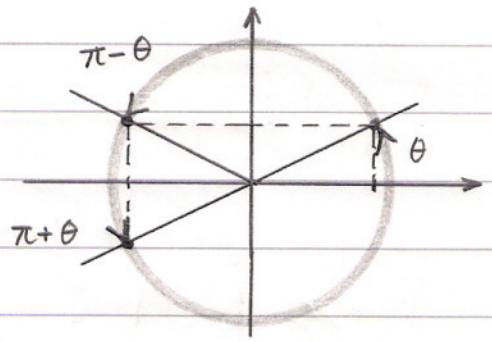
The square of the distance from $(0,0)$ to $(\cos(\theta), \sin(\theta))$ equals 1, so we have the fundamental identity :

$$\cos^2(\theta) + \sin^2(\theta) = 1.$$

By the following figures, we find other identities :



$$\cos(-\theta) = \cos(\theta)$$
$$\sin(-\theta) = -\sin(\theta)$$



$$\cos(\pi + \theta) = -\cos(\theta)$$
$$\cos(\pi - \theta) = -\cos(\theta)$$
$$\sin(\pi + \theta) = -\sin(\theta)$$
$$\sin(\pi - \theta) = \sin(\theta)$$

Note also that

$$\cos(2\pi + \theta) = \cos(\theta)$$
$$\sin(2\pi + \theta) = \sin(\theta)$$

One has addition and subtraction formulas for sine and cosine

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta),$$

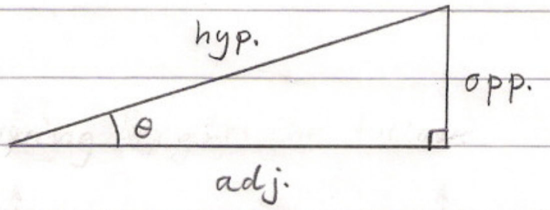
$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta).$$

If we choose $+$ and take $\alpha = \beta = \theta$, we get the double-angle formulas

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 1 - 2\sin^2(\theta)$$
$$= 2\cos^2(\theta) - 1,$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta).$$

Right triangles



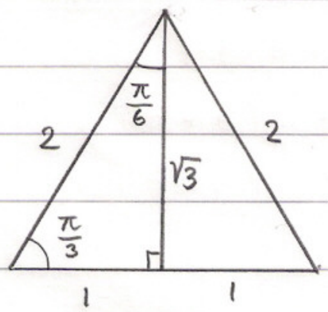
One can scale the triangle by $\frac{1}{hyp}$ and find:

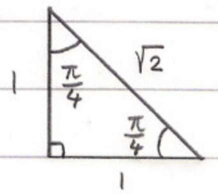
$$\cos(\theta) = \frac{adj}{hyp}, \quad \sin(\theta) = \frac{opp}{hyp}, \quad \tan(\theta) = \frac{opp}{adj},$$

$$\sec(\theta) = \frac{hyp}{adj}, \quad \csc(\theta) = \frac{hyp}{opp}, \quad \cot(\theta) = \frac{adj}{opp}.$$

Exact values

θ	$30°$ $\frac{\pi}{6}$	$45°$ $\frac{\pi}{4}$	$60°$ $\frac{\pi}{3}$
$\sin(\theta)$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos(\theta)$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan(\theta)$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

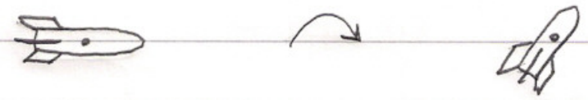




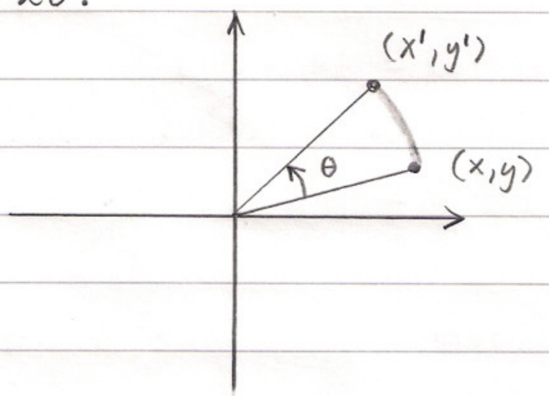
Applications

- Computer graphics

Rotation



2D:



$$x' = \cos(\theta)x - \sin(\theta)y$$
$$y' = \sin(\theta)x + \cos(\theta)y$$

3D: More complicated formulas involving sine and cosine.

- Sound

Sinusoid: $y(t) = A \cdot \sin(\omega t + \varphi)$

t time / s

ω angular frequency / $\frac{\text{rad}}{\text{s}}$

φ phase / rad

A amplitude / V