Exam in "Linear Algebra"
Test set B

First Year at The TEK-NAT Faculty and Health Faculty

It is allowed to use books, notes etc. It is not allowed to use any electronic devices such as pocket calculators, mobile phones or computers.

The listed percentages specify by which weight the individual exercises influence the total examination.

This exam set has two independent parts. Part I contains "regular exercises". Here it is important that you explain the idea behind the solution, and that you provide relevant intermediate results. Part II is "multiple choice" exercises. The answers for Part II must be given on these sheets.

Remember to write your full name (including middle names), student number together with the course number on each side of your answers. **Number each page and write the total number of pages on the front page** of the answers.

Good luck!

NAME: ______________________________

STUDENT NUMBER: ______________________________

COURSE NUMBER: ______________________________

Part I ("regular exercises")

Exercise 1:(5%) Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 10 & 20 \\ 30 & 40 \\ 50 & 60 \end{bmatrix}$$

1. Compute AB.
2. Find $B^T A^T$.

Exercise 2:(10%) Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

1. Reduce A to its reduced echelon.
2. Let R denote the reduced echelon form of A. Determine $\det R$.
3. Determine $\det A$.

Exercise 3:(10%) Consider the subspace

$$V = \left\{ \begin{bmatrix} r+2s \\ 2s \\ r+2s \end{bmatrix} \; : \; r \in \mathbf{R} \;\text{ og }\; s \in \mathbf{R} \right\} \subseteq \mathbf{R}^3.$$

1. Find a basis for V.
2. What is the dimensionen of V?
3. Is $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$ a basis for V? (Justify your answer.)
4. Is $\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$ a basis for V? (Justify your answer.)

Exercise 4:(10%) Let

$$A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}.$$

1. Find the eigenvalues of A.
2. Determine the associated eigenspaces.
3. Find matrices P and D, such that P is invertible, D is a diagonal matrix and $A = PDP^{-1}$ holds.

Exercise 5:(8%) Let

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}.$$

1. Explain why $\mathcal{B}$ is a basis for $\mathbb{R}^3$.

2. Let $\mathbf{v} \in \mathbb{R}^3$ be a vector with $\mathcal{B}$-coordinate-vector $[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Determine $\mathbf{v}$.

Let T be a linear operator $T : \mathbb{R}^3 \to \mathbb{R}^3$, where

$$\begin{aligned} T([\ 1 \quad 0 \quad 0\ ]^T) &= [\ 1 \quad 0 \quad 1\ ]^T \\ T([\ 0 \quad 1 \quad 0\ ]^T) &= [\ 0 \quad 1 \quad 0\ ]^T \\ T([\ 0 \quad 1 \quad 1\ ]^T) &= [\ 1 \quad 1 \quad 0\ ]^T. \end{aligned}$$

3. Find $T([\ 1 \quad 0 \quad 1\ ]^T)$ and $T([\ 1 \quad 1 \quad 0\ ]^T)$.

4. Write the matrix representation of T relative to $\mathcal{B}$.

Exercise 6:(10%)

Let W be the solution set for

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 0 \\ x_1 + x_2 - 3x_3 &= 0. \end{aligned}$$

Let $\mathbf{u} = \begin{bmatrix} 1 & 3 & -2 \end{bmatrix}^T$.

1. Determine the orthogonal projection matrix P_W.

2. Find $\mathbf{w} \in W$ and $\mathbf{z} \in W^{\perp}$, such that $\mathbf{u} = \mathbf{w} + \mathbf{z}$.

3. Find the distance from $\mathbf{u}$ to W.

Exercise 7:(9%) Let $C = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$.

Examine if C is invertible, and if so determine C^{-1}.

Exercise 8:(8%) Consider

$$S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} \right\}.$$

1. Is S linearly dependent? (Justify your answer.)
2. Is $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ in the span of S? (Justify your answer.)

Part II ("multiple choice" exercises)

Exercise 9:(5%)
There are given two vectors $\mathbf{u}, \mathbf{v} \in \mathbf{R}^3$, that are linearly *independent*. Let

$$H = \text{span}\{\mathbf{u}, \mathbf{v}\}$$

Mark the true statements below.

☐ H is a subspace of $\mathbf{R}^3$.
☐ H can be described as line in $\mathbf{R}^3$.
☐ There exists a $c \in \mathbf{R}$, such that either $\mathbf{v} = c\mathbf{u}$ or $\mathbf{u} = c\mathbf{v}$.

Exercise 10:(12%)
There is given two 3×3-matrices A and B. Furthermore, the determinants of the matrixes are $\det A = 2$ and $\det B = -3$. Answer the following questions based on this information:

a. Determine: $\det(-B)$

☐ 0 ☐ -3 ☐ 3 ☐ 2

b. Determine: $\det(A^2 B)$

☐ -12 ☐ -8 ☐ 12 ☐ 0

c. Determine: $\det(A(B^T)^2)$

☐ 1 ☐ -6 ☐ 18 ☐ 9

d. Determine: $\det(-AB)$

☐ 6 ☐ -6 ☐ -18 ☐ 2

Exercise 11:(8%) Answer the following 4 true/false questions:

a. Let W be a subspace of $\mathbf{R}^n$. If $\mathbf{w}$ is in both W og $W^\perp$ then $\mathbf{w} = \mathbf{0}$.

☐ True ☐ False

b. Consider the system of equations $A\mathbf{x} = \mathbf{0}$, there A is a 7×8-matrix. The system of equations has infinitely many solutions.

☐ True ☐ False

c. Let $\mathbf{u}, \mathbf{v} \in \mathbf{R}^n$. If $\mathbf{u} \cdot \mathbf{v} = 0$, then either $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$ holds.

☐ True ☐ False

d. Let W be a subspace of $\mathbf{R}^4$ with dimension 3. It is possible to select a number of vectors from W, which constitute a basis for $\mathbf{R}^4$.

☐ True ☐ False

Exercise 12:(5%) There are given three vectors $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbf{R}^3$, such that $\{\mathbf{u}, \mathbf{v}\}$ is linearly *independent*, while $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly *dependent*. Let

$$H = \text{span}\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$$

Mark the true statements below.

☐ It is always true that $\mathbf{v}$ can be written as a linear combination of $\mathbf{u}$ and $\mathbf{w}$.
☐ It is always true that $\{\mathbf{u}, \mathbf{w}\}$ is linearly independent.
☐ $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is not a basis for H.