Exam in "Linear Algebra"
Test set A

First Year at The TEK-NAT Faculty and Health Faculty

It is allowed to use books, notes etc. It is not allowed to use any electronic devices such as pocket calculators, mobile phones or computers.

The listed percentages specify by which weight the individual exercises influence the total examination.

This exam set has two independent parts. Part I contains "regular exercises". Here it is important that you explain the idea behind the solution, and that you provide relevant intermediate results. Part II is "multiple choice" exercises. The answers for Part II must be given on these sheets.

Remember to write your full name (including middle names), student number together with the course number on each side of your answers. **Number each page and write the total number of pages on the front page** of the answers.

Good luck!

NAME: ______________________________

STUDENT NUMBER: ______________________________

COURSE NUMBER: ______________________________

Part I ("regular exercises")

Exercise 1:(5%) Let

$$B = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}.$$

1. Find $\det B$.
2. Determine B^{-1}.
3. Determine $\left(B^T\right)^{-1}$.

Exercise 2:(8%) Consider the system of equations

$$\begin{array}{rrrcr} x_1 & +x_2 & +x_3 & = & 1 \\ & x_2 & -x_3 & = & 2 \\ x_1 & +2x_2 & & = & 3. \end{array}$$

1. List the coefficient matrix for the system of equations.
2. List the augmented matrix for system of equations.
3. Find the general solution to the system of equations.

Exercise 3:(8%) Let

$$A = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 4 & 1 & 1 \end{bmatrix}.$$

1. Find a basis for the column space associated with A.
2. Determine rank$A$ and nullity$A$.
3. How many vectors does a basis for the null space of A contain? (Justify your answer.)

Exercise 4:(9%) Let

$$C = \begin{bmatrix} -1 & 0 & 0 \\ 2 & 5 & 0 \\ 1 & -2 & -1 \end{bmatrix}.$$

1. Find the eigenvalues of C.
2. Determine a basis for each of the corresponding eigenspaces.

Exercise 5:(14%) Find the solution to the system of differential equations

$$\begin{aligned} y_1' &= y_1 + y_2 \\ y_2' &= 4y_1 + y_2 \end{aligned}$$

with initial conditions $y_1(0) = 15, y_2(0) = -10$.

Exercise 6:(10%)

$$S = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \\ 5 \end{bmatrix} \right\}$$

is a basis for the subspace $W \subseteq \mathbf{R}^4$.

1. Using the Gram-Schmidt procedure, find an orthogonal basis for W.
2. Determine hereafter an orthonormal basis for W.

Exercise 7:(8%) Let

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}.$$

1. Reduce A to its reduced echelon form using exactly two elementary row operations.
2. Let R be the reduced echelon form of A. Find elementary matrices E_1 and E_2, such that $R = E_2E_1A$ holds.
3. Find elementary matrices E_3 and E_4, such that $A = E_4E_3R$ holds.

Exercise 8:(8%) This exercise considers linear transformations from $\mathbf{R}^2$ to $\mathbf{R}^2$.

1. Write the matrix that rotates counter clock-wise by 90^o.
2. Write the matrix that reflects about the first axis.
3. Write the matrix, which corresponds the rotation in question 8.1 followed by the reflection in question 8.2.

Part II ("multiple choice" exercises)

Exercise 9:(7%)
Consider a $m \times n$ matrix A with the following properties:

1. A has 6 pivot columns.
2. There exists a $\mathbf{b} \in \mathbf{R}^m$, such that $A\mathbf{x} = \mathbf{b}$ is inconsistent.

Based on this information, determine the *minimal* values possible for m and n.

Specify the minimal value for m:

☐ 0	☐ 2	☐ 4	☐ 6	☐ 8	☐ 10
☐ 1	☐ 3	☐ 5	☐ 7	☐ 9	☐ 11

Specify the minimal value for n:

☐ 0	☐ 2	☐ 4	☐ 6	☐ 8	☐ 10
☐ 1	☐ 3	☐ 5	☐ 7	☐ 9	☐ 11

Exercise 10:(10%) Consider the matrix

$$A = \begin{bmatrix} -2 & -1 & -4 & 5 \\ 0 & -8 & 6 & -6 \\ 0 & 0 & -81 & 13 \\ 0 & 0 & 0 & -604 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Mark *all* the true statements below (note: every wrong answer *eliminates* one correct answer).

- ☐ A is invertible.
- ☐ The linear transformation induced by A is injective (one-to-one).
- ☐ A is on reduced echelon form.
- ☐ nullity $A = 0$.
- ☐ rank $A = 4$.
- ☐ A is on echelon form.
- ☐ nullity A + rank $A = 5$.
- ☐ The linear transformation induced by A is subjective (onto).
- ☐ For every $\mathbf{b} \in \mathbf{R}^5$ the system of equations $A\mathbf{x} = \mathbf{b}$ is consistent.
- ☐ A is a 4×5-matrix.

Exercise 11:(5%) There is given four vectors $\mathbf{x}, \mathbf{y}, \mathbf{u}, \mathbf{v} \in \mathbf{R}^3$, such that both $\{\mathbf{x}, \mathbf{y}\}$ and $\{\mathbf{u}, \mathbf{v}\}$ are linearly *independent*. Mark the true statement below.

- ☐ $\{\mathbf{x}, \mathbf{y}, \mathbf{u}, \mathbf{v}\}$ is always linearly independent.
- ☐ $\{\mathbf{x}, \mathbf{y}, \mathbf{u}, \mathbf{v}\}$ is never linearly independent.
- ☐ The information is not sufficient in order to determine whether $\{\mathbf{x}, \mathbf{y}, \mathbf{u}, \mathbf{v}\}$ is linearly independent.

Exercise 12:(8%) Answer the following 4 true/false exercises:

a. The standard matrix for an invertible linear transformation $T : \mathbf{R}^n \rightarrow \mathbf{R}^m$ is always a square matrix.

☐ True ☐ False

b. A linear transformation specified by a 4×5-matrix is never subjective (onto).

☐ True ☐ False

c. All invertible matrices can be diagonalized.

☐ True ☐ False

d. Let W be a subspace of $\mathbf{R}^6$ with dimension 4. Then it is always true that 4 linearly independent vectors in W constitute a basis for W.

☐ True ☐ False