Reexam in Discrete Mathematics

First Year at the Faculty of Engineering and Science and the Technical Faculty of IT and Design

August 15th, 2017, 9.00-13.00

This exam consists of 11 numbered pages with 15 problems. All the problems are "multiple choice" problems. **The answers must be given on these sheets.**

It is allowed to use books, notes, photocopies etc. It is not allowed to use any electronic devices such as pocket calculators, mobile phones or computers.

The listed percentages specify by which weight the individual problems influence the total examination.

Remember to write your full name (including middle names) together with your student number below.

NAME: ____________________

STUDENT NUMBER: ____________________

There is only one correct answer to each question.

Problem 1 (8 %)

1. How many rows appear in a truth table of the compound proposition

$$(p \lor r) \land (r \to q) \lor s$$

☐ 1 ☐ 2 ☐ 4 ☐ 8 ☐ 12 ☐ 16

2. Are the propositions $\neg(p \land q)$ and $\neg p \lor q$ equivalent?

☐ Yes ☐ No

3. Is the compound proposition $(p \to q) \leftrightarrow (\neg p \lor q)$ a tautology?

☐ Yes ☐ No

4. Are the propositions $\neg(p \to q)$ and $(p \land \neg q)$ equivalent?

☐ Yes ☐ No

Problem 2 (4 %)

Which rule of inference is used in the following argument:

"I will go to London or Paris. I will not go to London. Therefore, I will go to Paris."

☐ Conjuction
☐ Modus tollens
☐ Modus ponens
☐ Hypothetical syllogism
☐ Disjunctive syllogism

Problem 3 (10 %)

Let $A = \{1, 2, 5, \{6\}\}$, and $B = \{\emptyset, 1, \{2\}, 6, 7\}$, be sets.

1. Is $\{2\} \subseteq B$?

   ☐ Yes ☐ No

2. What is the cardinality of $A \cap B$?

   ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7

3. What is the cardinality of $A \cup B$?

   ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9

4. What is the cardinality of the power set $\mathcal{P}(A)$?

   ☐ 0 ☐ 2 ☐ 6 ☐ 8 ☐ 12 ☐ 16 ☐ 22 ☐ 32

5. What is the cardinality of $A \times B$?

   ☐ 0 ☐ 2 ☐ 4 ☐ 8 ☐ 10 ☐ 20 ☐ 200 ☐ 2^9

Problem 4 (9 %)

Let

$$f(x) = x^3 \log(x^2) + \log(x^{10} + 1) + x^3,$$

for $x > 0$.

Answer the following true/false problems.

1. $f(x)$ is $O(x^{10})$ ☐ True ☐ False
2. $f(x)$ is $O(x^3 \log x)$ ☐ True ☐ False
3. $f(x)$ is $O(x^3)$ ☐ True ☐ False
4. $f(x)$ is $\Omega(x^{10})$ ☐ True ☐ False
5. $f(x)$ is $\Omega(x^3 \log x)$ ☐ True ☐ False
6. $f(x)$ is $\Omega(x^3)$ ☐ True ☐ False
7. $f(x)$ is $\Theta(x^{10})$ ☐ True ☐ False
8. $f(x)$ is $\Theta(x^3 \log x)$ ☐ True ☐ False
9. $f(x)$ is $\Theta(x^3)$ ☐ True ☐ False

Problem 5 (4 %)

$(5 + 55 + 101)(576 \cdot 555 + 10000000002)$ **mod** 5 is equal to

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

Problem 6 (7 %)

What is the inverse of 7 modulo 53?

☐ 14 ☐ 15 ☐ 16 ☐ 17 ☐ 37 ☐ 38 ☐ 39 ☐ 43 ☐ 45

Problem 7 (5 %)

Consider the following algorithm:

procedure *Alg*(n: positive integer)
$a := 1$
$b := 1$
for $i := 1$ **to** n
  for $j := 1$ **to** 1000000
    for $k := 1$ **to** n
      $a := a \cdot b$
  $b := 3 \cdot b$
return a

The number of multiplications used by this algorithm is

☐ $O(n)$ ☐ $\Theta(n^2 \log n)$ ☐ $O(n^2)$ ☐ $\Theta(n^3)$ ☐ $O(n \log n)$

Problem 8 (6 %)

Let $(2x - y)^5 = ax^5 + bx^4y + cx^3y^2 + dx^2y^3 + exy^4 + fy^5$, where a, b, c, d, e, f are integers.

1. One has that a is equal to

☐ -1 ☐ 1 ☐ 8 ☐ 5 ☐ 16 ☐ 32 ☐ 100

2. One has that c is equal to

☐ -80 ☐ -40 ☐ -20 ☐ -10 ☐ 10 ☐ 20 ☐ 40 ☐ 80

3. One has that d is equal to

☐ -80 ☐ -40 ☐ -20 ☐ -10 ☐ 10 ☐ 20 ☐ 40 ☐ 80

Problem 9 (4 %)

Consider the following set of integers

$$S = \{x \mid -63 \leq x \leq 126 \ \wedge \ x \equiv 5 \pmod{7} \ \wedge \ x \equiv 5 \pmod{9}\}$$

How many integers are there in S?

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 63 ☐ 126 ☐ 189

Problem 10 (6 %)

Consider the following linear homogeneous recurrence relation

$$a_n = a_{n-1} + 6a_{n-2}.$$

Which of the following is the solution of this recurrence relation (α_1 and α_2 are constants)?

☐ $a_n = \alpha_1(-2)^n + \alpha_2(-3)^n$
☐ $a_n = \alpha_1 + \alpha_2 3^n$
☐ $a_n = \alpha_1 + \alpha_2 2^n$
☐ $a_n = \alpha_1(-2)^n + \alpha_2 3^n$
☐ $a_n = \alpha_1 2^n + \alpha_2(-3)^n$

Problem 11 (6 %)

A set of integers S is defined recursively by

- $0 \in S$, $5 \in S$ and $6 \in S$
- if $a \in S$ and $b \in S$ then $a + b$ is also in S.

Answer the following questions about S

1. What is the largest integer not contained in S?

   ☐ 9 ☐ 13 ☐ 14 ☐ 18 ☐ 19 ☐ 20 ☐ 21 ☐ 22 ☐ 23

2. S consists of all numbers of the form $5r + 6t$ where r and t are integers satisfying . . .
   What is the correct continuation?

   ☐ $r > 0 \ \wedge \ s > 0$
   ☐ $r \geq 0 \ \wedge \ s \geq 0$
   ☐ $r \geq 0 \ \wedge \ s \geq 0 \ \wedge \ r + t > 0$
   ☐ $r \geq 1 \ \vee \ s \geq 0$
   ☐ $r > 6 \ \vee \ s \geq 5$

Problem 12 (10 %)

Let $A = \{1, 2, 3, 4\}$ be a set. Consider the following three relations on A

$$R = \{(1,1), (1,2), (1,3), (2,4), (3,2)\}$$

$$S = \{(1,4), (1,3), (2,3), (3,1), (4,1)\}$$

$$T = \{(1,2), (2,3), (3,4), (2,1)\}$$

1. Answer the following true/false problems.

R is reflexive	☐ True	☐ False
R is symmetric	☐ True	☐ False
R is antisymmetric	☐ True	☐ False
R is transitive	☐ True	☐ False
$(2,1) \in S \circ R$	☐ True	☐ False
$(2,1) \in R \circ S$	☐ True	☐ False

2. How many pairs (a, b) are there in the symmetric closure of S?

   ☐ 0 ☐ 1 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10

3. How many pairs (a, b) are there in the transitive closure of T?

   ☐ 0 ☐ 1 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10

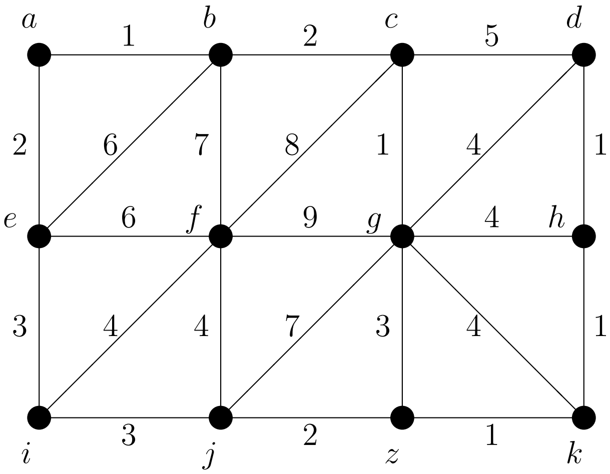

Figure 1: The graph G, considered in Problems 13 and 14.

Problem 13 (10 %)

In this problem we use Dijkstra's algorithm (see Figure 2 on page 11) on the graph G in Figure 1.

1. What is the length of the shortest path from a to z (found by Dijkstra's algorithm)?

   ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10 ☐ 11 ☐ 12 ☐ 13

2. In what order are vertices added to the set S ?

   ☐ a, b, c, g, z
   ☐ a, b, e, c, g, i, j, z
   ☐ a, b, e, c, g, i, z
   ☐ a, b, c, g, i, z
   ☐ a, b, e, c, g, z
   ☐ a, b, c, e, i, g, z
   ☐ $a, b, c, d, e, f, g, h, i, j, k, z$

Problem 14 (5 %)

What is the weight of a minimum spanning tree of the graph G in Figure 1.

☐ 19 ☐ 20 ☐ 21 ☐ 22 ☐ 23 ☐ 24 ☐ 25 ☐ 26 ☐ 27

Problem 15 (6 %)

Let T be a full binary tree with 6 leaves.

1. How many vertices are there in T?

   ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10 ☐ 11 ☐ 12

2. What is the least possible height of T?

   ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7

3. What is the largest possible height of T?

   ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7

procedure *Dijkstra*(G: weighted connected simple graph, with
    all weights positive)
$\{G$ has vertices $a = v_0, v_1, \ldots, v_n = z$ and lengths $w(v_i, v_j)$
    where $w(v_i, v_j) = \infty$ if $\{v_i, v_j\}$ is not an edge in $G\}$
for $i := 1$ **to** n
    $L(v_i) := \infty$
$L(a) := 0$
$S := \emptyset$
{the labels are now initialized so that the label of a is 0 and all
    other labels are ∞, and S is the empty set}
while $z \notin S$
    $u :=$ a vertex not in S with $L(u)$ minimal
    $S := S \cup \{u\}$
    for all vertices v not in S
        if $L(u) + w(u, v) < L(v)$ **then** $L(v) := L(u) + w(u, v)$
        {this adds a vertex to S with minimal label and updates the
        labels of vertices not in S}
return $L(z)$ $\{L(z) =$ length of a shortest path from a to $z\}$

Figure 2: