Self-study session 1

In this self-study Matlab is used as an important aid to solve the exercises. As an introduction to Matlab you can watch the screencasts 2 and 3 as available on the first math website: `http://first.math.aau.dk/eng/software/matlab/`

You can use the available `Self-study-1.m` file to save the required commands for each of the exercises and to run (that is to execute) them. Remarks concerning Matlab coding:

1. command `disp('....')` writes text to the screen.

2. use explanatory names for the output variables .

3. commands stated between %% and %% can be executed by left-clicking on it (thereby activating this "section") and then pressing the ***Run section* command button**.

4. Remark: If you immediately write text after %% without a space as in: %%Exercise-1..., then the commmand (as opening or ending a section) is not recognized.

To start with this self-study,

1. save the `Self-study-1.m` file on your hard-drive,

2. make a copy of it that you e.g. name by adding your initials to it (and potentially a version number), like `Self-study-1-initials-v1.m`,

3. then start Matlab and open the file in Matlab.

4. Now you click on the first section (below *%% Chapter 1 Exercise 1*) and then press the "Run section" command button) (and confirm "add the path to the current one ").

5. ...thereby the commands defined in this first section are executed and the output is written to the command window.

Roughly the time it will take you to go through the Tasks 1 to 4 is respectively 45, 45, 30 and 15 minutes.

Task 1

Solve exercises 1, 2, 4, 5, 7 and 8 from pages 90-91 in the book. Remarks concerning the `Self-study-1.m` file:

- Exercise-1: Solution to 1a is given as an example.

- Exercise-2: The matrices A, B and vector $\mathbf{v}$ are already defined and the solutions to 2a, b, and c are given as an example.

- Exercise-4: is premade you can skip through the steps by running sequentially the various code sections, each defining one Gaussian elimination cycle.

- Exercise-5: Exercise 5a is premade as an example.

- Exercise-7: Some of the to be applied commands are given for 7a and 7b.

Task 2

a) Solve exercises 94, 95, 96 on page 55 by finding the reduced echelon form of each of the matrices by the `rref` command of Matlab and then solve the system of linear equations analytical (by hand) from the reduced echelon form (see for instance example 3 on page 34 and 35).

b) Now apply the following alternative option to solve the same exercises as in a) by applying the Matlab-commands:

`linsolve:` gives a solution to a consistent system of linear equations. (Note: for an inconsistent system the command will give an *approximate solution.*

`null(A):` returns a set of linearly independent vectors $\mathbf{x}$ that are solutions to $A\mathbf{x} = \mathbf{0}$, where $\mathbf{0}$ is the zero vector. The number n of linearly independent vectors $\mathbf{x}$ for which $A\mathbf{x} = \mathbf{0}$, is called the nullity of A, where n can also be equal to zero.

As an example we consider Practice Problem 1 at page 46 of the book. We type in the coefficient matrix A and the vector $\mathbf{b}$ on the right hand side of the equation.

```
>> A = [1 -1 -3 1 -1 ; -2 2 6 0 -6 ; 3 -2 -8 3 -5] <enter>
A =
     1    -1    -3     1    -1
```

```
   -2     2     6     0    -6
    3    -2    -8     3    -5

>> b = -[2 ; 6 ; 7] <enter>
b =
    -2
    -6
    -7
```

Now by `linsolve` we find a solution of $A\mathbf{x} = \mathbf{b}$:

```
>> w = linsolve(A, b) <enter>
w =
         0
    1.8750
   -0.3750
         0
    1.2500
```

We can check that this indeed a solution by verifying that $A\mathbf{w} - \mathbf{b} = \mathbf{0}$:

```
>> A*w - b <enter>
ans =
   1.0e-14 *

    0.1110
    0.2665
    0.0888
```

The difference between $A\mathbf{w}$ and $\mathbf{b}$ is less than 10^{-14} (so not exactly zero but this is due to (extremely small) numerical error(!))

The general (full) solution for $\mathbf{x}$ for which $A\mathbf{x} = \mathbf{b}$, are all those vectors $\mathbf{x}$ generated by adding to the solution as given by the `linsolve` command—whose output is a single column vector which we denoted by `w`—any linear combination of the solution(s) of the homogeneous equation $A\mathbf{x} = \mathbf{0}$—as obtained by applying the `null`) command. That is with

```
>> null(A)
ans =
   -0.4584    0.6799
    0.4932   -0.1239
   -0.1412    0.4120
```

```
0.7040      0.5761
0.1760      0.1440
```

and denoting the first column vector above as $\mathbf{u}$ and the 2nd column vector by $\mathbf{v}$, then all the vectors $\mathbf{x} = \mathbf{w} + s\mathbf{u} + t\mathbf{v}$ for any $s, t \in \mathbb{R}$ are solutions to the matrix equation $A\mathbf{x} = \mathbf{b}$.

Verify the correctness of the latter statement by for some values of s and t create $\mathbf{x} = \mathbf{w} + s\mathbf{u} + t\mathbf{v}$ for the exercises as mentioned in (a) and checking that then indeed $A\mathbf{x} - \mathbf{b}$ is zero (that is uptill numerical error). Use again the `.m` file in which this is done as an example for the here above discussed Practice problem 1.

Task 3

Next, solve Exercises 3 and 6 on pages 90–91, define the rotation matrix A_θ yourself in Matlab (that is do not use the `rotdeg` function). Note that the input in the $\sin(x)$ and $\cos(x)$ functions in Matlab should be in radians and not in degrees.

Task 4

Solve Exercises 94, 95, 96 at page 55, by using the `rref` command to obtain the reduced echelon form of the augmented matrices.