

Self-study session 1, Linear algebra

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The below self-study is supported by screencasts 2 and 3. It might be a good idea to watch the screencasts before continuing. When needed during the exercise solving you can watch them again.

The first task is to solve the following exercises from pages 90-91 in the book: 1, 2, 4, 5, 7, 8.

We next consider Exercises 94, 95, 96 at page 55. The authors of the book intend you to use only MATLAB to do the Gaussian elimination part (the command `rref`). It is then the authors intention that you should solve the system of linear equations by hand from the row-reduced echelon form. In the following you will be taken through an alternative option. The idea is that you should use this alternative option when solving Exercises 94, 95, 96 at page 55.

We shall need the following commands:

MATLAB-commands

`linsolve` gives a single solution to a consistent system of linear equations; for a non-consistent system it *approximates* a solution.

`null(A)` returns a set v_1, v_2, \dots, v_n of linearly independent vectors that are solutions to $Ax = 0$, where 0 is the zero vector. The number of vectors, n , equals the nullity of A .

Let us consider Pratic Problem 1 at page 46 of the book. We first type in the coefficient matrix for the equation and the right side of the equation.

```
>> A = [1 -1 -3 1 -1 ; -2 2 6 0 -6 ; 3 -2 -8 3 -5]
```

```
A =
```

```
    1    -1    -3     1    -1
   -2     2     6     0    -6
    3    -2    -8     3    -5
```

```
>> b = -[2 ; 6 ; 7]
```

```
b =
```

```
   -2
   -6
   -7
```

With `linsolve` we find a single (that is, not all) solution of $Ax = b$:

```
>> x = linsolve(A, b)
```

```
x =
    0
    1.8750
   -0.3750
    0
    1.2500
```

To check if what we found is actually a solution we must investigate if $Ax - b = 0$ holds.

```
>> A*x - b

ans =

    1.0e-14 *
    0.1110
    0.2665
    0.0888
```

Apparently the difference between Ax and b is a number close to 10^{-14} , which we interpret as a numerical error.

The idea is that we know that the general solution consists of a particular solution – that is, any solution to the system (which is found by applying `linsolve`) – plus the general solution to the homogeneous equation $A\vec{x} = \vec{0}$ (which is found by applying `null`).

```
>> null(A)

ans =

   -0.4584    0.6799
    0.4932   -0.1239
   -0.1412    0.4120
    0.7040    0.5761
    0.1760    0.1440
```

In the above answer, name the first vector \vec{u} and the latter vector \vec{v} . For various values of the variables s, t check that $\vec{x} + s\vec{u} + t\vec{v}$ is a solution to the matrix equation.

Solve the three exercises from page 55 using the same method that we used above for the case of Practice Problem 1.

Next, solve Exercises 3, 6 at pages 90-91.

Finally, solve Exercises 94, 95, 96 at page 55 using the method prescribed in the book. That is, using the `rref` command. This method is also described at the webpage

<https://se.mathworks.com/help/matlab/ref/rref.html>