Self-study session 2, Calculus

First year mathematics for the technology and science programmes
Aalborg University

The purpose of this self-study session is to provide a perspective on some of the key concepts and approaches presented in E&P Chapter 12. Feel free to use Matlab or Maple[1] for symbolic computations.

Part I: Tangent plane and optimization

A function is for all $(x, y) \in \mathbb{R}^2$ defined by

$$f(x, y) = 4xy^2 + 2x^2 + 6y^2 + 10.$$

The surface $\mathcal{F}$ is the graph for $f(x, y)$. That is, $\mathcal{F}$ is given by the equation $z = f(x, y)$.

a) Determine the equation for the tangent plane of the surface $\mathcal{F}$ at the point $P(-1, 2, f(-1, 2))$.

b) Determine those points $(x, y, f(x, y))$ at which the tangent planes of $\mathcal{F}$ are parallel with the xy-plane.

c) A region R is determined by $y \geq 0$, $x \leq 0$ and $y^2 \leq x + 6$. Sketch the region R.

d) Determine the maximal and minimal values of $f(x, y)$ in the region R.

Part II: Gradient vector, chain rule and implicitly defined functions

A function F is defined by

$$F(x, y, z) = x^2 \cos y + 2y \cos x + 3z - \sin z.$$

a) Determine the gradient vector ∇F to F at the point $P(0, 0, 0)$.

b) Determine the directional derivative to F at the point P in the direction given by the vector $\mathbf{v} = [2, 2, -1]^\top$.

c) The funtion $f(x, y)$ is defined implicitly by $F(x, y, f(x, y)) = 0$. Determine $f(0, 0)$, $f_x(0, 0)$ and $f_y(0, 0)$.

[1] Partial derivatives can easily be calculated symbolically in Matlab and Maple. As an example here is how to compute f_x for the function $f(x, y) = x^2y^3$:

```
≫ syms x y
≫ f = x^2*y^3
≫ diff(f,x)
```

In Maple use: `diff(x^2*y^3,x)`.