Session 1 & 2
Solutions to problems related to the exercises

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Converting degrees to radians

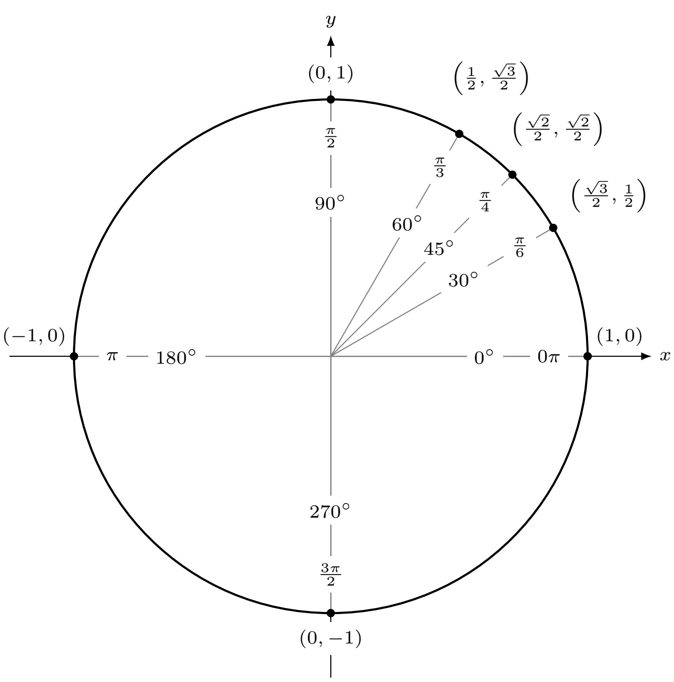

Figure S1.1: The unit circle with some angles illustrated.

In some exercises, you need to convert degrees to radians or vice versa. To do this, you need the "exchange rate." As 180° corresponds to π radians, namely half a round on the unit circle, the exchange rate from degrees to radians must be must be π radians per 180°, or $\frac{\pi}{180°}$. Thus, if we want to convert 45° to radians, we get $45° \cdot \frac{\pi}{180°} = \frac{45°}{180°}\pi = \frac{1}{4}\pi = \frac{\pi}{4}$. To go from radians to from radians to degrees, we just have to find the opposite exchange rate: 180° per π radians, or $\frac{180°}{\pi}$. Thus, if we want to find $\frac{\pi}{5}$ radians in degrees, we just calculate $\frac{\pi}{5} \cdot \frac{180°}{\pi} = \frac{180°}{5} = 36°$.

Finding solutions to trigonometric equations

We now want to illustrate how to find solutions to trigonometric equations by solving $\cos(x) = 1$. Remember that if θ is the (signed) angle between the positive x-axis and a half-line starting in $(0, 0)$, then the half-line intersects the unit circle in the point $(\cos(\theta), \sin(\theta))$, i.e. cosine is the first coordinate of the point of intersection. So, when is $\cos(x) = 1$? As can be seen in Figure S1.1, the only point on the unit circle whose first coordinate is 1 is the point $(1, 0)$. Which angle does the half-line starting in $(0, 0)$ and passing through $(1, 0)$ have to the positive x-axis? Again, the answer can be read of in Figure S1.1. The answer is 0 radians (or $0°$). This means that $\cos(x) = 1$ has the solution $x = 0$. Are there any other solutions? Well, what happens if you to an integer multiple of 2π around the unit circle – in positive or negative direction? You end up where you started! Since this covers all possible ways of ending up in $(1, 0)$, all solutions to $\cos(x) = 1$ are of the form $x = 0 + 2n\pi$, where n is an integer. We also write this as

$$\cos(x) = 1 \quad \Leftrightarrow \quad x = 2n\pi,\ n \in \mathbb{Z},$$

where $\mathbb{Z}$ is the standard notation for the set of integers. Note that $x = 0$ is included as $0 = 2 \cdot 0 \cdot \pi$ and 0 is an integer.

A hint to the proof of the cosine addition formula

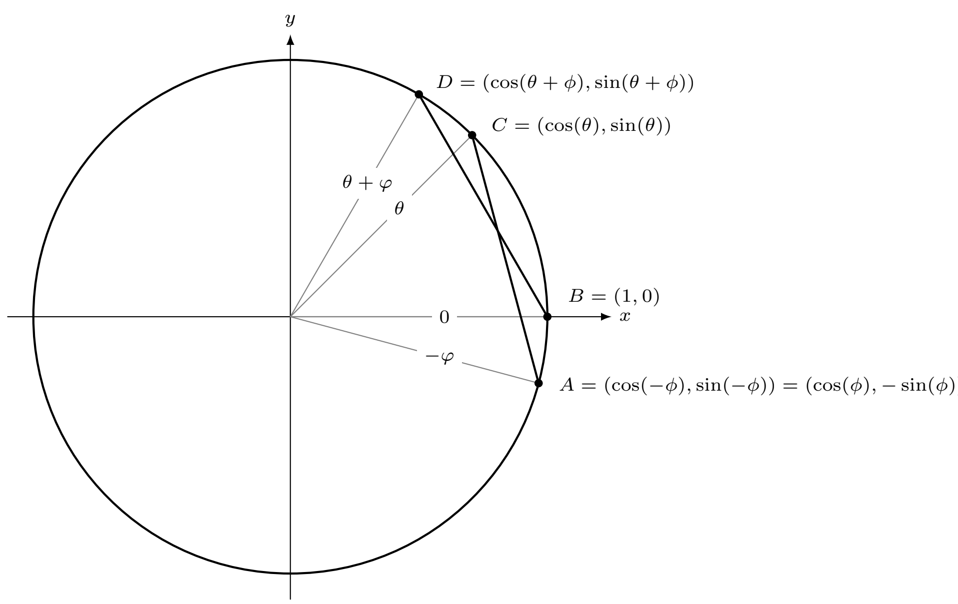

Figure S1.2: A sketch of the situation in Problem C.41.

If one wants to compare the lengths $|AC|$ and $|BD|$ of the two line segments AC and BD, respectively, one first has to know how to calculate the length of a line segment between two points. To illustrate how to do this, we will now calculate the length $|AB|$ of the line segment AB. First, one subtracts the coordinates of A from the coordinates of B: $(1, 0) - (\cos(\phi), -\sin(\phi)) = (1 - \cos(\phi), -(-\sin(\phi))) = (1 - \cos(\phi), \sin(\phi))$. Then we take the square root of the sum of squares of the two coordinates to get the length: $|AB| = \sqrt{(1 - \cos(\phi))^2 + \sin^2(\phi)}$.