TABLE OF INTEGRALS

ELEMENTARY FORMS

1. $\int u\,dv = uv - \int v\,du$

2. $\int u^n\,du = \frac{1}{n+1}u^{n+1} + C \quad \text{if } n \neq -1$

3. $\int \frac{du}{u} = \ln|u| + C$

4. $\int e^u\,du = e^u + C$

5. $\int a^u\,du = \frac{a^u}{\ln a} + C$

6. $\int \sin u\,du = -\cos u + C$

7. $\int \cos u\,du = \sin u + C$

8. $\int \sec^2 u\,du = \tan u + C$

9. $\int \csc^2 u\,du = -\cot u + C$

10. $\int \sec u\,\tan u\,du = \sec u + C$

11. $\int \csc u\,\cot u\,du = -\csc u + C$

12. $\int \tan u\,du = \ln|\sec u| + C$

13. $\int \cot u\,du = -\ln|\csc u| + C$

14. $\int \sec u\,du = \ln|\sec u + \tan u| + C$

15. $\int \csc u\,du = -\ln|\csc u + \cot u| + C$

16. $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\frac{u}{a} + C$

17. $\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$

18. $\int \frac{du}{a^2 - u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C$

19. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C$

TRIGONOMETRIC FORMS

20. $\int \sin^2 u\,du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$

21. $\int \cos^2 u\,du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$

22. $\int \tan^2 u\,du = \tan u - u + C$

23. $\int \cot^2 u\,du = -\cot u - u + C$

24. $\int \sin^3 u\,du = -\frac{1}{3}(2 + \sin^2 u)\cos u + C$

25. $\int \cos^3 u\,du = \frac{1}{3}(2 + \cos^2 u)\sin u + C$

26. $\int \tan^3 u\,du = \frac{1}{2}\tan^2 u + \ln|\cos u| + C$

27. $\int \cot^3 u\,du = -\frac{1}{2}\cot^2 u - \ln|\sin u| + C$

28. $\int \sec^3 u\,du = \frac{1}{2}\sec u\,\tan u + \frac{1}{2}\ln|\sec u + \tan u| + C$

29. $\int \csc^3 u\,du = -\frac{1}{2}\csc u\,\cot u - \frac{1}{2}\ln|\csc u + \cot u| + C$

30. $\int \sin au\,\sin bu\,du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C \quad \text{if } a^2 \neq b^2$

31. $\int \cos au\,\cos bu\,du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C \quad \text{if } a^2 \neq b^2$

32. $\int \sin au\,\cos bu\,du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C \quad \text{if } a^2 \neq b^2$

33. $\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1} u\,\cos u + \frac{n-1}{n}\int \sin^{n-2} u\,du$

34. $\int \cos^n u\,du = \frac{1}{n}\cos^{n-1} u\,\sin u + \frac{n-1}{n}\int \cos^{n-2} u\,du$

35. $\int \tan^n u\,du = \frac{1}{n-1}\tan^{n-1} u - \int \tan^{n-2} u\,du \quad \text{if } n \neq 1$

36. $\int \cot^n u\,du = -\frac{1}{n-1}\cot^{n-1} u - \int \cot^{n-2} u\,du \quad \text{if } n \neq 1$

37. $\int \sec^n u\,du = \frac{1}{n-1}\sec^{n-2} u\,\tan u + \frac{n-2}{n-1}\int \sec^{n-2} u\,du \quad \text{if } n \neq 1$

38. $\int \csc^n u\,du = -\frac{1}{n-1}\csc^{n-2} u\,\cot u + \frac{n-2}{n-1}\int \csc^{n-2} u\,du \quad \text{if } n \neq 1$

39a. $\int \sin^n u\,\cos^m u\,du = -\frac{\sin^{n-1} u\,\cos^{m+1} u}{n+m} + \frac{n-1}{n+m}\int \sin^{n-2} u\,\cos^m u\,du \text{ if } n \neq -m$

39b. $\int \sin^n u\,\cos^m u\,du = -\frac{\sin^{n+1} u\,\cos^{m-1} u}{n+m} + \frac{m-1}{n+m}\int \sin^n u\,\cos^{m-2} u\,du \text{ if } m \neq -n$

40. $\int u\sin u\,du = \sin u - u\cos u + C$

41. $\int u\cos u\,du = \cos u + u\sin u + C$

42. $\int u^n \sin u\,du = -u^n\cos u + n\int u^{n-1}\cos u\,du$

43. $\int u^n \cos u\,du = u^n\sin u - n\int u^{n-1}\sin u\,du$

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Table of Integrals continues from front cover

FORMS INVOLVING $\sqrt{u^2 \pm a^2}$

44. $\displaystyle\int \sqrt{u^2 \pm a^2}\,du = \frac{u}{2}\sqrt{u^2 \pm a^2} \pm \frac{a^2}{2}\ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

45. $\displaystyle\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

46. $\displaystyle\int \frac{\sqrt{u^2 + a^2}}{u}\,du = \sqrt{u^2 + a^2} - a\ln\left(\frac{a + \sqrt{u^2 + a^2}}{u}\right) + C$

47. $\displaystyle\int \frac{\sqrt{u^2 - a^2}}{u}\,du = \sqrt{u^2 - a^2} - a\sec^{-1}\frac{u}{a} + C$

48. $\displaystyle\int u^2\sqrt{u^2 \pm a^2}\,du = \frac{u}{8}(2u^2 \pm a^2)\sqrt{u^2 \pm a^2} - \frac{a^4}{8}\ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

49. $\displaystyle\int \frac{u^2\,du}{\sqrt{u^2 \pm a^2}} = \frac{u}{2}\sqrt{u^2 \pm a^2} \mp \frac{a^2}{2}\ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

50. $\displaystyle\int \frac{du}{u^2\sqrt{u^2 \pm a^2}} = \mp\frac{\sqrt{u^2 \pm a^2}}{a^2 u} + C$

51. $\displaystyle\int \frac{\sqrt{u^2 \pm a^2}}{u^2}\,du = -\frac{\sqrt{u^2 \pm a^2}}{u} + \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

52. $\displaystyle\int \frac{du}{(u^2 \pm a^2)^{3/2}} = \pm\frac{u}{a^2\sqrt{u^2 \pm a^2}} + C$

53. $\displaystyle\int (u^2 \pm a^2)^{3/2}\,du = \frac{u}{8}(2u^2 \pm 5a^2)\sqrt{u^2 \pm a^2} + \frac{3a^4}{8}\ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

FORMS INVOLVING $\sqrt{a^2 - u^2}$

54. $\displaystyle\int \sqrt{a^2 - u^2}\,du = \frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$

55. $\displaystyle\int \frac{\sqrt{a^2 - u^2}}{u}\,du = \sqrt{a^2 - u^2} - a\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

56. $\displaystyle\int \frac{u^2\,du}{\sqrt{a^2 - u^2}} = -\frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$

57. $\displaystyle\int u^2\sqrt{a^2 - u^2}\,du = \frac{u}{8}(2u^2 - a^2)\sqrt{a^2 - u^2} + \frac{a^4}{8}\sin^{-1}\frac{u}{a} + C$

58. $\displaystyle\int \frac{du}{u^2\sqrt{a^2 - u^2}} = -\frac{\sqrt{a^2 - u^2}}{a^2 u} + C$

59. $\displaystyle\int \frac{\sqrt{a^2 - u^2}}{u^2} = -\frac{\sqrt{a^2 - u^2}}{u} - \sin^{-1}\frac{u}{a} + C$

60. $\displaystyle\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a}\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

61. $\displaystyle\int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2 - u^2}} + C$

62. $\displaystyle\int (a^2 - u^2)^{3/2}\,du = \frac{u}{8}(5a^2 - 2u^2)\sqrt{a^2 - u^2} + \frac{3a^4}{8}\sin^{-1}\frac{u}{a} + C$

EXPONENTIAL AND LOGARITHMIC FORMS

63. $\displaystyle\int ue^u\,du = (u - 1)e^u + C$

64. $\displaystyle\int u^n e^u\,du = u^n e^u - n\int u^{n-1}e^u\,du$

65. $\displaystyle\int \ln u\,du = u\ln u - u + C$

66. $\displaystyle\int u^n \ln u\,du = \frac{u^{n+1}}{n+1}\ln u - \frac{u^{n+1}}{(n+1)^2} + C$

67. $\displaystyle\int e^{au}\sin bu\,du = \frac{e^{au}}{a^2 + b^2}(a\sin bu - b\cos bu) + C$

68. $\displaystyle\int e^{au}\cos bu\,du = \frac{e^{au}}{a^2 + b^2}(a\cos bu + b\sin bu) + C$

INVERSE TRIGONOMETRIC FORMS

69. $\displaystyle\int \sin^{-1} u\,du = u\sin^{-1} u + \sqrt{1 - u^2} + C$

70. $\displaystyle\int \tan^{-1} u\,du = u\tan^{-1} u - \frac{1}{2}\ln(1 + u^2) + C$

71. $\displaystyle\int \sec^{-1} u\,du = u\sec^{-1} u - \ln\left|u + \sqrt{u^2 - 1}\right| + C$

72. $\displaystyle\int u\sin^{-1} u\,du = \frac{1}{4}(2u^2 - 1)\sin^{-1} u + \frac{u}{4}\sqrt{1 - u^2} + C$

73. $\displaystyle\int u\tan^{-1} u\,du = \frac{1}{2}(u^2 + 1)\tan^{-1} u - \frac{u}{2} + C$

74. $\displaystyle\int u\sec^{-1} u\,du = \frac{u^2}{2}\sec^{-1} u - \frac{1}{2}\sqrt{u^2 - 1} + C$

75. $\displaystyle\int u^n \sin^{-1} u\,du = \frac{u^{n+1}}{n+1}\sin^{-1} u - \frac{1}{n+1}\int \frac{u^{n+1}}{\sqrt{1 - u^2}}\,du \quad \text{if } n \neq -1$

76. $\displaystyle\int u^n \tan^{-1} u\,du = \frac{u^{n+1}}{n+1}\tan^{-1} u - \frac{1}{n+1}\int \frac{u^{n+1}}{1 + u^2}\,du \quad \text{if } n \neq -1$

77. $\displaystyle\int u^n \sec^{-1} u\,du = \frac{u^{n+1}}{n+1}\sec^{-1} u - \frac{1}{n+1}\int \frac{u^{n+1}}{\sqrt{u^2 - 1}}\,du \quad \text{if } n \neq -1$

HYPERBOLIC FORMS

78. $\int \sinh u\,du = \cosh u + C$

79. $\int \cosh u\,du = \sinh u + C$

80. $\int \tanh u\,du = \ln(\cosh u) + C$

81. $\int \coth u\,du = \ln|\sinh u| + C$

82. $\int \operatorname{sech} u\,du = \tan^{-1}|\sinh u| + C$

83. $\int \operatorname{csch} u\,du = \ln\left|\tanh\frac{u}{2}\right| + C$

84. $\int \sinh^2 u\,du = \frac{1}{4}\sinh 2u - \frac{u}{2} + C$

85. $\int \cosh^2 u\,du = \frac{1}{4}\sinh 2u + \frac{u}{2} + C$

86. $\int \tanh^2 u\,du = u - \tanh u + C$

87. $\int \coth^2 u\,du = u - \coth u + C$

88. $\int \operatorname{sech}^2 u\,du = \tanh u + C$

89. $\int \operatorname{csch}^2 u\,du = -\coth u + C$

90. $\int \operatorname{sech} u \tanh u\,du = -\operatorname{sech} u + C$

91. $\int \operatorname{csch} u \coth u\,du = -\operatorname{csch} u + C$

MISCELLANEOUS ALGEBRAIC FORMS

92. $\int u(au+b)^{-1}\,du = \frac{u}{a} - \frac{b}{a^2}\ln|au+b| + C$

93. $\int u(au+b)^{-2}\,du = \frac{1}{a^2}\left(\ln|au+b| + \frac{b}{au+b}\right) + C$

94. $\int u(au+b)^n\,du = \frac{(au+b)^{n+1}}{a^2}\left(\frac{au+b}{n+2} - \frac{b}{n+1}\right) + C$ if $n \neq -1, -2$

95. $\int \frac{du}{(a^2 \pm u^2)^n} = \frac{1}{2a^2(n-1)}\left(\frac{u}{(a^2 \pm u^2)^{n-1}} + (2n-3)\int \frac{du}{(a^2 \pm u^2)^{n-1}}\right)$ if $n \neq 1$

96. $\int u\sqrt{au+b}\,du = \frac{2}{15a^2}(3au - 2b)(au+b)^{3/2} + C$

97. $\int u^n\sqrt{au+b}\,du = \frac{2}{a(2n+3)}\left(u^n(au+b)^{3/2} - nb\int u^{n-1}\sqrt{au+b}\,du\right)$

98. $\int \frac{u\,du}{\sqrt{au+b}} = \frac{2}{3a^2}(au - 2b)\sqrt{au+b} + C$

99. $\int \frac{u^n\,du}{\sqrt{au+b}} = \frac{2}{a(2n+1)}\left(u^n\sqrt{au+b} - nb\int \frac{u^{n-1}\,du}{\sqrt{au+b}}\right)$

100a. $\int \frac{du}{u\sqrt{au+b}} = \frac{1}{\sqrt{b}}\ln\left|\frac{\sqrt{au+b}-\sqrt{b}}{\sqrt{au+b}+\sqrt{b}}\right| + C$ if $b > 0$

100b. $\int \frac{du}{u\sqrt{au+b}} = \frac{2}{\sqrt{-b}}\tan^{-1}\sqrt{\frac{au+b}{-b}} + C$ if $b < 0$

101. $\int \frac{du}{u^n\sqrt{au+b}} = -\frac{\sqrt{au+b}}{b(n-1)u^{n-1}} - \frac{(2n-3)a}{(2n-2)b}\int \frac{du}{u^{n-1}\sqrt{au+b}}$ if $n \neq 1$

102. $\int \sqrt{2au-u^2}\,du = \frac{u-a}{2}\sqrt{2au-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u-a}{a} + C$

103. $\int \frac{du}{\sqrt{2au-u^2}} = \sin^{-1}\frac{u-a}{a} + C$

104. $\int u^n\sqrt{2au-u^2}\,du = -\frac{u^{n-1}(2au-u^2)^{3/2}}{n+2} + \frac{(2n+1)a}{n+2}\int u^{n-1}\sqrt{2au-u^2}\,du$

105. $\int \frac{u^n\,du}{\sqrt{2au-u^2}} = -\frac{u^{n-1}}{n}\sqrt{2au-u^2} + \frac{(2n-1)a}{n}\int \frac{u^{n-1}\,du}{\sqrt{2au-u^2}}$

106. $\int \frac{\sqrt{2au-u^2}}{u}\,du = \sqrt{2au-u^2} + a\sin^{-1}\frac{u-a}{a} + C$

107. $\int \frac{\sqrt{2au-u^2}}{u^n}\,du = \frac{(2au-u^2)^{3/2}}{(3-2n)au^n} + \frac{n-3}{(2n-3)a}\int \frac{\sqrt{2au-u^2}}{u^{n-1}}\,du$

108. $\int \frac{du}{u^n\sqrt{2au-u^2}} = \frac{\sqrt{2au-u^2}}{a(1-2n)u^n} + \frac{n-1}{(2n-1)a}\int \frac{du}{u^{n-1}\sqrt{2au-u^2}}$

109. $\int \left(\sqrt{2au-u^2}\right)^n\,du = \frac{u-a}{n+1}(2au-u^2)^{n/2} + \frac{na^2}{n+1}\int \left(\sqrt{2au-u^2}\right)^{n-2}\,du$ if $n > 2$

110. $\int \frac{du}{\left(\sqrt{2au-u^2}\right)^n} = \frac{u-a}{(n-2)a^2}\left(\sqrt{2au-u^2}\right)^{2-n} + \frac{n-3}{(n-2)a^2}\int \frac{du}{\left(\sqrt{2au-u^2}\right)^{n-2}}$ if $n > 2$

DEFINITE INTEGRALS

111. $\int_0^\infty u^n e^{-u}\,du = \Gamma(n+1) = n!$ $(n \geq 0)$

112. $\int_0^\infty e^{-au^2}\,du = \frac{1}{2}\sqrt{\frac{\pi}{a}}$ $(a > 0)$

113. $\int_0^{\pi/2}\sin^n u\,du = \int_0^{\pi/2}\cos^n u\,du = \begin{cases} \frac{1\cdot 3\cdot 5\cdots(n-1)}{2\cdot 4\cdot 6\cdots n}\cdot\frac{\pi}{2} & \text{if } n \text{ is an even integer and } n \geq 2 \\ \frac{2\cdot 4\cdot 6\cdots(n-1)}{3\cdot 5\cdot 7\cdots n} & \text{if } n \text{ is an odd integer and } n \geq 3 \end{cases}$