Miniproject 3

The present miniproject is supported by screencast 6. It might be a good idea to watch the screencast before continuing. When needed during the exercise solving you can watch it again.

First, read from the middle of page 339 till the middle of page 341 in [SIF]. We now plot the solution from the middle of page 341. That is, we plot

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 10\exp(3t) \\ 12\exp(4t) \\ 15\exp(5t) \end{bmatrix} \tag{1}$$

using two different methods.

<u>Plot method 1</u>

Download the file p340comet.m. Place it in your MATLAB library. Write "p340comet" in the command line and you will see an animation of (1) emerging. Close the plot window (important). Open p340comet.m in a text editor. Change "t=0:0.01:2" to "t=0:0.001:2". This means that the plot is now supported by 10 times as many data points. Save the file and execute it with the command p340comet. Close the plot again (important). Make experiments with other choices of starting point, ending point and number of data points. Do this by modifying the content of p340comet.m. As an example you can replace "t=0:0.001:2" with t=0:0.001:5".

<u>Plot method 2</u>

Download the file p340plot.m. Execute it by typing in the command p340plot. As before modify "t=0:0.01:2" and run the command again. It is crucial that you remember to close down the plot window every time. In one of the plots choose "Insert->X Label". Type "y1" in the box. Do the same for y2 and y3. If you like you can now save the plot. You can also play with rotating the figure.

Next, continue reading in [SIF]. Read until the start of Practice Problem 2 at the middle of page 342.

Lets plot the solution in Example 3

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 20\exp(t) + 100\exp(5t) \\ -60\exp(t) + 100\exp(5t) \end{bmatrix}. \tag{2}$$

Download the files ex3comet.m and ex3plot.m. Notice, that line 2 of ex3comet.m now calls "comet" rather than "comet3". Similarly, ex3plot.m now calls "plot" rather than "plot3". Execute the commands and play with the values of t (as before). Why is the plot so boring?

We next solve Example 3 using MATLAB. The calculations appear very different from the result in the book. This is because MATLAB chooses another ordering of the eigenvalues and because MATLAB choose eigenvectors of length 1. The general solution looks different from the one in the book (but is actually the same). The solution corresponding to the given initial conditions is exactly as in the book.

Download the file ex3en.m and place it in the MATLAB catalog. If you run the command ex3en you will see the following:

```
>> ex3en

A =

     4     1
     3     2


y0 =

   120
    40


P =

    0.7071   -0.3162
    0.7071    0.9487


D =

    5.0000         0
         0    1.0000


ygen =

   (2^(1/2)*a*exp(5*t))/2 - (10^(1/2)*b*exp(t))/10
 (2^(1/2)*a*exp(5*t))/2 + (3*10^(1/2)*b*exp(t))/10


ygenrounded =

 0.71*a*exp(5.0*t) - 0.32*b*exp(t)
 0.71*a*exp(5.0*t) + 0.95*b*exp(t)
```

```
y =

 100*exp(5*t) + 20*exp(t)
 100*exp(5*t) - 60*exp(t)


yrounded =

 100.0*exp(5.0*t) + 20.0*exp(t)
 100.0*exp(5.0*t) - 60.0*exp(t)
```

The expression "ygen" is not easy to read. We therefore translate it into decimal numbers which we round of. The result is "ygenrounded". In the above case the expression "y" is readable and a translation to decimal numbers which are rounded of is not really any help ("yrounded" is less nice than "y").

Open the file ex3en.m and read the comments. On the basis of this you should be able to reuse the content of the file to solve the exercises below. For every exercise you should produce a new .m file.

A few general comments: If you have three variables you should define $arb = [a;b;c]$ and you must remember to change $[t;t]$ to $[t;t;t]$. If you arrive at the "$2/exp(3*t)$" then it corresponds to "$2*exp(-3*t)$". If you get the answer "8.6e-17" then it means 0.000000000000000086, which you of course read as 0.

1. Using MATLAB solve the following system of differential equations:

$$\begin{cases} y_1' &= -3y_1 + y_2 + y_3 \\ y_2' &= 8y_1 - 2y_2 - 4y_3 \\ y_3' &= -10y_1 + 2y_2 + 4y_3 \end{cases}$$

   under the initial condition

$$\begin{cases} y_1(0) &= 2 \\ y_2(0) &= -3 \\ y_3(0) &= 6 \end{cases}$$

   Plot the solution.

2. Extend the text of Exercises 47, 49, 51 at page 350 with initial conditions of your own choice. Solve the exercise in MATLAB and plot the solutions.
3. Solve Exercises 53, 55, 57 at page 351.
4. Solve Exercise 89 at page 353.

Finally, solve MATLAB exercises from pages 288–289 in the text book:

- Exercise. 1, 2, 4 (only questions a, b, d), 5, 6.