Calculus miniproject 2: "Partial derivatives"

First year of study at the Faculty of Engineering and Science
and
the Faculty of Medicine

The purpose of this mini project is to provide a perspective on some of the key concepts and approaches presented in E&P Chapter 12. You are welcome to use Matlab or Maple [1] for symbolic computations.

Part I: Tangentplane and optimization

A function is for all $(x, y) \in R^2$ defined by

$$f(x, y) = 4xy^2 + 2x^2 + 6y^2 + 10$$

The surface F is the graph for $f(x, y)$. I.e. F is given by the equation $z = f(x, y)$.

a) Determine the equation for the tangent plane of the surface F at the point $P\big(-1,2, f(-1,2)\big)$.
b) Determine those points $\big(x, y, f(x, y)\big)$ at which the tangent planes of F are parallel to the xy-plane.
c) A region R is determined by $y \geq 0, x \leq 0$ and $y^2 \leq x + 6$. Sketch the region R.
d) Determine the maximal and minimal values of $f(x, y)$ in the region R.

Part II: Gradientvector, Chainrule and implitly defined functions.

A function F is defined by

$$F(x, y, z) = x^2 \cos y + 2y \cos x + 3z - \sin z$$

a) Determine the gradient vector ∇F to F at the point $P(0,0,0)$.
b) Determine the directional derivative to Fat P in the direction determined by the vector $\vec{\boldsymbol{v}} = [2,2,-1]^\top$.
c) The function $f(x, y)$ is implicitly defined by $F\big(x, y, f(x, y)\big) = 0$. Determine $f(0,0), f_x(0,0)$ and $f_y(0,0)$.

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[1] Partial derivatives can easily be calculated symbolically in Matlab and Maple. As an example here is how to compute f_x for the function $f(x,y) = x^2\, y^3$:

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≫ syms x y
≫ f = x^2*y^3
≫ diff( f, x )
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In Maple use

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> diff(x^2*y^3, x).
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