Test-exam in Mathematics for Multimedia Applications

First Year at The Faculty of Engineering and Science

March 2016

This exam set consists of 9 pages with 14 problems. For each question a number of points are indicated. The total number of points equals 100.

It is allowed to use books, notes, photocopies etc. It is **not allowed** to use **any electronic devices** such as pocket calculators, mobile phones or computers.

The exam set has two independent parts.

- Part I contains "essay problems". Here it is important that you explain the idea behind the solution, and that you provide relevant intermediate results.
- Part II contains "multiple choice" problems. **The answers of Part II must be given on these sheets.**

Remember to write your full name (including middle names) together with your student number below. Also write name and student number on each page of you solutions of the essay problems and number these pages. Indicate the total number of extra sheets on the first page.

Good luck!

NAME: ________________________________

STUDENT NUMBER: ________________________________

Part I (Essay-problems)

Problem 1 (9 points)

(a) (2 points). Prove that the following identity holds:

$$\sin(4x) = 2\sin(2x)\cos(2x).$$

Hint: Use the double angle formula for sine.

(b) (4 points). Prove the trigonometric identity

$$\sin(4x) = 4(\sin(x)\cos^3(x) - \sin^3(x)\cos(x)).$$

Hint: Use double angle formulas for sine and cosine.

(c) (3 points). Describe all solutions of the equation

$$\sin(x)\cos^3(x) - \sin^3(x)\cos(x) = 0.$$

Problem 2 (11 points)

Consider the following system of linear equations:

$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ x_1 + 2x_2 - 3x_3 &= 1 \\ -x_1 + x_2 - 9x_3 &= -4 \end{aligned}$$

(a) (2 points). Find the augmented matrix of the system.

(b) (5 points). Find the reduced row echelon form of the augmented matrix.

(c) (4 points). Write down the general solution of the system.

Part II (Multiple-choice problems)

Problem 3 (4 points)

A function is given by

$$f(x) = \sin(3x+2) + e^{3x}.$$

Mark the correct expression for its derivative $f'(x)$.

- ☐ $\cos(3x+2) + e^{3x}$
- ☐ $\sin(3x+2) + \frac{1}{3}e^{3x}$
- ☐ $\cos(3x) + 3e^{3x}$
- ☐ $3\cos(3x+2) + 3e^{3x}$
- ☐ $3\cos(3x+2) + 3e^{2x}$
- ☐ $-\cos(3x+2) + \frac{1}{3}e^{3x}$

Problem 4 (6 points)

A function is defined by

$$g(x) = \ln(x^2 - 4x + 1).$$

The graph of the function has a horizontal tangent at a point. What is the x-coordinate of that point?

- ☐ $\ln(5)$
- ☐ 2
- ☐ 0
- ☐ 1
- ☐ $\frac{1}{4}$
- ☐ e

Problem 5 (5 points)

What is the value of the limit

$$\lim_{h \to 0} \frac{(a+h)^3 - a^3}{h}$$

where a is a constant?

- ☐ ∞
- ☐ $2a$
- ☐ 1
- ☐ $3a^2$
- ☐ a^2
- ☐ a^3

Problem 6 (3 points)

The sum

$$\sum_{i=1}^{5}(5i - i^2)$$

is equal to

- ☐ 10
- ☐ 20
- ☐ 5
- ☐ -9
- ☐ 7
- ☐ 100

Problem 7 (5 points)

The sum

$$\sum_{i=1}^{10}(6i^2 + 18i)$$

is equal to

- ☐ 3300
- ☐ 5000
- ☐ 2500
- ☐ 10000
- ☐ 3000
- ☐ 700

Problem 8 (5 points)

The integral

$$\int_1^3 (x + \frac{1}{x})dx$$

is equal to

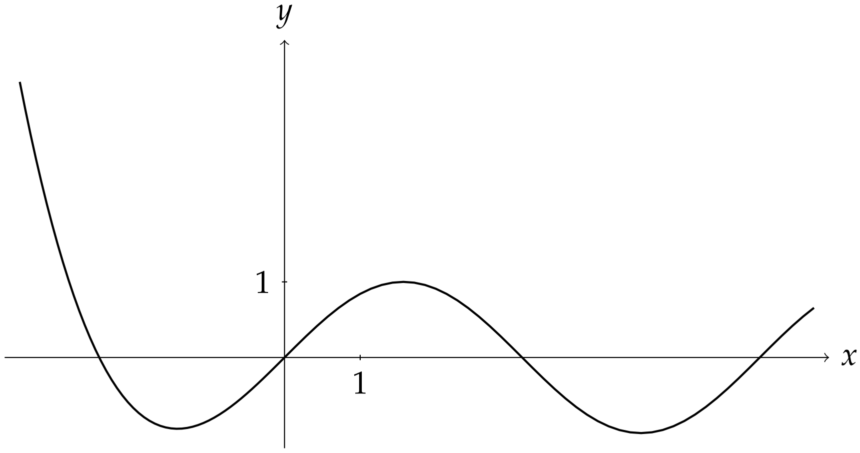
- ☐ $\frac{3}{2}$
- ☐ $e^2 + 2$
- ☐ 1
- ☐ $\ln(3) + 8$
- ☐ $\ln(3) + 4$
- ☐ $\ln(2) + 8$

Problem 9 (7 points)

A function is defined by

$$f(x) = \begin{cases} x - \frac{1}{6}x^3, & x < 0, \\ \sin(x), & x \geq 0. \end{cases}$$

The graph of the function looks as follows:



(a) (3 ponts). The integral $\int_0^{\pi} f(x)dx$ equals

- ☐ $\frac{\pi}{2}$
- ☐ 1.8
- ☐ 1
- ☐ 3
- ☐ 2
- ☐ -1

(b) (3 points). The integral $\int_{-1}^{0} f(x)dx$ equals

- ☐ $-\frac{1}{2}$
- ☐ $-\frac{1}{4}$
- ☐ $-\frac{11}{24}$
- ☐ -2
- ☐ $-\frac{\pi}{4}$
- ☐ -1

(c) (1 point). The integral $\int_0^{2\pi} f(x)dx$ equals

- ☐ $\frac{\pi}{2}$
- ☐ $\frac{1}{3}$
- ☐ 0
- ☐ 3
- ☐ 1
- ☐ -1

Problem 10 (16 ponts)

Three points in 3D-space are given by

$$P = (1, -1, 1), \quad Q = (2, 1, 3), \quad R = (5, 2, 1).$$

In consequence, we have the following two vectors

$$\overrightarrow{PQ} = (1, 2, 2), \quad \overrightarrow{PR} = (4, 3, 0).$$

Mark the correct answers below.

(a) (2 points). The coordinates of the vector $\overrightarrow{QR}$ are

- ☐ $(1, 3, -2)$
- ☐ $(2, -3, 0)$
- ☐ $(-3, -1, 2)$
- ☐ $(3, 1, -2)$

(b) (3 points). The line through P and Q has parametric equation

- ☐ $(x, y, z) = (1, 2, 2) + t(4, 3, 0)$
- ☐ $(x, y, z) = (2, 1, 3) + t(4, 3, 0)$
- ☐ $(x, y, z) = (1, -1, 1) + t(1, 2, 2)$
- ☐ $(x, y, z) = (1, 0, 0) + t(1, 1, 1)$

(c) (3 points). The angle between the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ is

- ☐ $\cos^{-1}(\frac{1}{5})$
- ☐ $\cos^{-1}(\frac{2}{3})$
- ☐ $\frac{\pi}{4}$
- ☐ $\frac{\pi}{2}$

A computation shows that $\overrightarrow{PQ} \times \overrightarrow{PR} = (-6, 8, -5)$.

(d) (2 points). What is the area of the triangle with vertices P, Q and R?

- ☐ 5
- ☐ 10
- ☐ $\frac{11}{2}$
- ☐ $\frac{5\sqrt{5}}{2}$

(e) (3 points). Which of the equations below describes the plane through P, Q and R?

- ☐ $-6x + 8y - 5z = 3$
- ☐ $6x - 8y + 5z = 19$
- ☐ $5x + 2y + z = 12$
- ☐ $x + 2y + 2z = 1$

(f) (3 points). Let $\vec{a} = (2, 3, 1)$ and $\vec{b} = (1, -1, 3)$. Then $\vec{a} \times \vec{b}$ equals

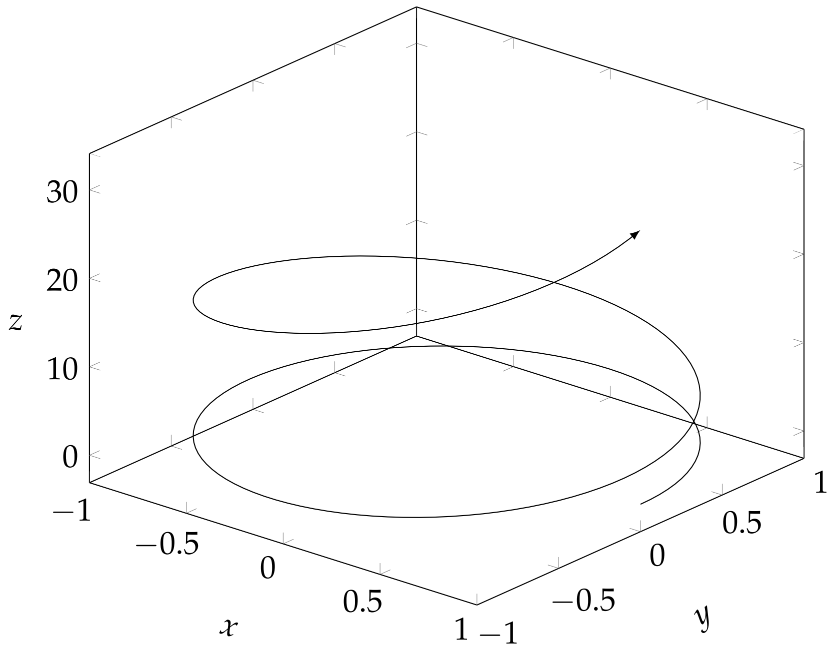
- ☐ $(-2, 1, 1)$
- ☐ $(4, -2, -2)$
- ☐ $(10, -5, -5)$
- ☐ $(10, -1, 3)$

Problem 11 (12 points)

The position vector of a moving particle in 3D-space is given by

$$\vec{r}(t) = \big(\cos(4t), \sin(4t), t^3\big).$$

Here is a plot of the motion curve when the time t runs from 0 to π:



(a) (2 points). At time $t = 0$ the particle is located at the point

- ☐ $(1, 1, 1)$ ☐ $(0, 1, 0)$
- ☐ $(1, 0, 0)$ ☐ $(1, 0, \frac{1}{2})$

(b) (3 points). The velocity vector of the particle at time $t = 0$ equals

- ☐ $\big(-\sin(4), \cos(4), 0\big)$ ☐ $\big(0, 1, 0\big)$
- ☐ $\big(-1, 1, 0\big)$ ☐ $\big(0, 4, 0\big)$

(c) (4 points). The speed $v(t)$ of the particle as a function of time t is

- ☐ $\sqrt{16 + 9t^4}$ ☐ $\sqrt{2 + 9t^2}$
- ☐ $\sqrt{1 + 9t^4}$ ☐ $\sqrt{4 + 9t^2}$

(d) (3 points). The acceleration vector of the particle at time $t = 0$ equals

- ☐ $(-8, -8, 0)$ ☐ $(8, 8, 0)$
- ☐ $(-4, -4, 3)$ ☐ $(-16, 0, 0)$

Problem 12 (7 points)

Two matrices are given by

$$A = \begin{bmatrix} 1 & -1 & 0 & 7 \\ 2 & 0 & 1 & -1 \\ 1 & -4 & 5 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \\ 3 & 3 & 2 \\ 1 & 0 & 1 \end{bmatrix}.$$

Mark the correct statements below.

(a) (1 point). The matrix product AB has size

- ☐ 4×4
- ☐ 3×4
- ☐ 3×3
- ☐ 4×3

(b) (3 points). Entry $(2, 3)$ of the matrix product AB, i.e. $[AB]_{23}$, equals

- ☐ 1
- ☐ 0
- ☐ -1
- ☐ 16

(c) (3 points). Put $C = A^T + B$. Entry $(3, 2)$ of matrix C, i.e. $[C]_{32}$, equals

- ☐ -5
- ☐ 1
- ☐ 0
- ☐ 4

Problem 13 (7 points)

A matrix is defined as

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & -1 & 1 \\ 1 & 1 & 3 \end{bmatrix}.$$

Mark the correct statement below.

- ☐ A is invertible and entry $(3, 3)$ of its inverse, i.e. $[A^{-1}]_{33}$ equals 3.
- ☐ A is invertible and entry $(3, 3)$ of its inverse, i.e. $[A^{-1}]_{33}$ equals -1.
- ☐ A is invertible and entry $(3, 3)$ of its inverse, i.e. $[A^{-1}]_{33}$ equals 1.
- ☐ A is not invertible.

Problem 14 (3 points)

Let A be the following matrix:

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

The associated matrix transformation

$$T : \mathcal{R}^2 \to \mathcal{R}^2; \quad T(\vec{x}) = A\vec{x}$$

describes a rotation about the origin. What is the rotation angle?

- ☐ $\frac{\pi}{2}$
- ☐ $\frac{\pi}{3}$
- ☐ π
- ☐ $-\frac{\pi}{3}$
- ☐ $-\frac{\pi}{2}$
- ☐ $\frac{\pi}{6}$