

Gram-Schmidt

$$S = \{a_1, \dots, a_k\} \text{ lin. unabh.}$$



$$v_1 = a_1$$

$$v_2 = a_2 - \frac{a_2 \cdot v_1}{\|v_1\|^2} v_1$$

$$v_3 = a_3 - \frac{a_3 \cdot v_1}{\|v_1\|^2} v_1 - \frac{a_3 \cdot v_2}{\|v_2\|^2} v_2$$

$$u_i = \frac{v_i}{\|v_i\|} \quad a_1 \quad a_2 \quad a_3$$

$$S = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \\ 5 \end{bmatrix} \right\}$$

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix} - \frac{6}{6} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} 3 \\ 1 \\ 1 \\ 5 \end{bmatrix} - \frac{12}{6} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix} - \frac{8}{6} \begin{bmatrix} 0 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ -1 \\ 3 \end{bmatrix}$$

Gram-Schmidt

$$S = \{a_1, \dots, a_k\} \text{ lin. unabh.}$$

$$V_1 = a_1$$

$$V_2 = a_2 - \frac{a_2 \cdot V_1}{\|V_1\|^2} V_1$$

$$V_3 = a_3 - \frac{a_3 \cdot V_1}{\|V_1\|^2} V_1 - \frac{a_3 \cdot V_2}{\|V_2\|^2} V_2$$



$$u_i = \frac{V_i}{\|V_i\|}, a_1, a_2, a_3$$

$$S = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \\ 5 \end{bmatrix} \right\}$$

$$V_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}, V_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix} - \frac{6}{6} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

$$V_3 = \begin{bmatrix} 3 \\ 1 \\ 1 \\ 5 \end{bmatrix} - \frac{12}{6} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix} - \frac{8}{6} \begin{bmatrix} 0 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 3 \end{bmatrix}$$

$$\frac{V}{\|V_1\|} = \frac{1}{\sqrt{6}} V_1$$

$$\frac{V_2}{\|V_2\|} = \frac{1}{\sqrt{6}} V_2$$

$$\frac{V_3}{\|V_3\|} = \frac{1}{\sqrt{\frac{4}{3}}} V_3 = \frac{\sqrt{3}}{2} V_3$$