(Practice)Exam in Linear Algebra

May 2016

First Year at The Faculties of Engineering and Science and of Health

This test has 10 pages and 16 multiple-choice problems. In two-sided print. It is allowed to use books, notes, photocopies etc. It is not allowed to use any electronic devices such as pocket calculators, mobile phones or computers.

The listed percentages specify by which weight the individual problems influence the total examination.

Remember to write your full name (including middle names) together with your student number on each side of your answers. Number each page. Write the total number of pages and the page number on each page of the answers.

NAME: _____

STUDENT NUMBER: _____

Problem 1 (7%)

Let

$$W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\} \text{ and } A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 2 & 3 & 0 \end{bmatrix}.$$

Answer the following 3 questions about W and A.

a. The dimension of W equals

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

b. The dimension of $W^{\perp}$ equals

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

c. The rank of A is

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

Problem 2 (8%)

Let

$$A = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 3 \end{bmatrix}.$$

a. The value of entry $(1,3)$ in A^{-1}, i.e. $(A^{-1})_{13}$, is:

☐ 0 ☐ 1 ☐ 1/3 ☐ 1/4 ☐ 1/5

b. The value of $\det(A^{-1})$ is

☐ 0 ☐ -1 ☐ -1/3 ☐ -1/4 ☐ 1/5

Part II (Multiple-choice problems)

Problem 3 (5%)

Let R be the row reduced echelon form of the matrix

$$A = \begin{bmatrix} 1 & -2 & 0 & 2 \\ 2 & 2 & 2 & 1 \end{bmatrix}$$

Specify the value of R_{24}:

☐ 0 ☐ $-\frac{1}{2}$ ☐ $\frac{1}{3}$ ☐ 2 ☐ $-\frac{3}{8}$

Problem 4 (10%).

Consider the matrix

$$A = \begin{bmatrix} 1 & 3 & 1 & 4 \\ 0 & -5 & 7 & 2 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 6 \end{bmatrix}.$$

Mark all correct statements below (notice: every incorrect mark cancels a correct one).

☐ A is not invertible.
☐ The linear transformation induced by A is injectiove (one-to-one).
☐ A is in row-echelon form.
☐ nullity $A = 1$.
☐ rank $A = 3$.
☐ nullity A + rank $A = 6$.
☐ The number 0 is an eigenvalue of A.
☐ A is in reduce row-echelon form.
☐ There is a vector $\mathbf{b} \in \mathbf{R}^4$, such that $A\mathbf{x} = \mathbf{b}$ is not consistent.
☐ $\det A = 0$.

Problem 5 (8%)

Given two 3×3-matrices A og B. Suppose that $\det A = -3$ and that B is an orthogonal matrix with $\det(B) > 0$. Answer the following questions:

a. Specify $\det B$:

☐ 0 ☐ -2 ☐ 1 ☐ 0.1

b. Specify $\det(AB)$:

☐ -2 ☐ 2 ☐ -3 ☐ 0

c. Specify $\det(-A)$:

☐ 1 ☐ -3 ☐ 1/2 ☐ 3

Problem 6 (7%).

Answer the following 4 true/false questions:

a. Let W be a subspace of $\mathbf{R}^6$ having dimension 4. Then $\dim(W^\perp) = 2$.

☐ True ☐ False

b. There exists a surjective (onto) linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^3$.

☐ True ☐ False

c. Suppose Q is a 4×4 ortogonal matrix. Then Q^5 is an ortogonal matrix.

☐ True ☐ False

d. A 3×3 matrix A with eigenvalues $1, 2$ and -3 is *both* invertible and diagonalizable.

☐ True ☐ False

Problem 7 (5%)

Which of the following statements are true (notice: every incorrect mark cancels a correct one):

☐ Any orthonormal set in $\mathbf{R}^n$ is a basis for $\mathbf{R}^n$, $n > 1$.
☐ The vectors in an orthonormal set in $\mathbf{R}^n$ are linearly independent.
☐ The vectors in an orthonormal set in $\mathbf{R}^n$ span $\mathbf{R}^n$.
☐ The number of vectors in an orthonormal set in $\mathbf{R}^n$ is at most n.

Problem 8 (5%)

Let C be given by

$$C = \begin{bmatrix} 2 & -3 & 0 & 3 \\ 0 & -2 & 2 & -3 \\ 0 & 3 & -2 & -3 \\ 0 & -1 & 1 & -2 \end{bmatrix}.$$

Then $\det(C)$ is:

☐ 2 ☐ -3 ☐ 0 ☐ 4 ☐ -2/5

Problem 9 (5%)

Let

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \\ -1 & 1 & -3 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}.$$

Answer the following 2 true/false questions:

i. The vector $\mathbf{b}$ is contained in $\mathrm{Col}(A)$.

☐ True ☐ False

ii. The vector $\mathbf{b}$ is contained in $\mathrm{Nul}(A)$.

☐ True ☐ False

Problem 10 (5%)

The following basis is given

$$\mathbf{b}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \quad \text{and} \quad \mathbf{b}_3 = \begin{bmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix},$$

for $\mathbf{R}^3$. Denote $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ and consider the vector

$$\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}.$$

Answer the following two questions:

i. $\mathcal{B}$ is an orthonormal basis $\mathbf{R}^3$.

☐ True ☐ False

ii. The third coordinate of $[\mathbf{v}]_{\mathcal{B}}$ is given by:

☐ $-\sqrt{2}$ ☐ 3 ☐ $\frac{1}{2}$ ☐ $-\frac{1}{\sqrt{3}}$ ☐ $\frac{1}{\sqrt{2}}$

Problem 11 (8%)

The row-echelon reduced form of the matrix

$$A = \begin{bmatrix} -2 & 2 & 3 & 1 & -3 & -2 \\ 0 & 2 & 0 & 0 & 1 & -1 \\ -1 & 0 & 1 & -3 & -2 & 2 \end{bmatrix}$$

is given by

$$R = \begin{bmatrix} 1 & 0 & 0 & 10 & 2 & -7 \\ 0 & 1 & 0 & 0 & 1/2 & -1/2 \\ 0 & 0 & 1 & 7 & 0 & -5 \end{bmatrix}.$$

Answer the following 4 questions about A:

a. The number of pivots of A is:

☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6

b. The number of free variables in the system of equations $A\mathbf{x} = \mathbf{0}$ is:

☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6

c. Let T be the linear transformation $T : \mathbf{R}^6 \to \mathbf{R}^d$ given by $T(\mathbf{x}) = A\mathbf{x}$. The number d is:

☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6

d. The linear transformation $T(\mathbf{x}) = A\mathbf{x}$, $\mathbf{x} \in \mathbf{R}^6$, is surjective (onto).

☐ True ☐ False

Problem 12 (5%)

Consider the system of equations

$$\begin{cases} x_1 + x_3 = 3 \\ x_1 - x_2 - x_3 = 1 \\ -x_1 + x_2 = 4 \end{cases}$$

This system has (mark only one statement):

- ☐ No solution
- ☐ An infinite number of solutions
- ☐ A uniquely determined solution
- ☐ None of the above statements apply.

Problem 13 (5%)

Consider the matrix

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 2 & -1 & 0 \end{bmatrix}$$

Which of the following statements hold true (mark only one statement):

- ☐ A's columns are linearly dependent
- ☐ $\det(A) = 1$
- ☐ A is not invertible
- ☐ None of the above statements apply.

Problem 14 (7%)

The number of linearly independent eigenvectors of the matrix

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

is given by:

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5

Problem 15 (5%)

Consider the matrix product AB, where

$$A = \begin{bmatrix} -2 & 0 & -3 & 2 & -2 & -3 \\ 1 & -2 & -2 & -2 & 3 & -3 \\ 2 & 2 & 1 & 0 & -3 & 1 \\ 3 & 3 & -1 & 2 & 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 & -2 \\ -3 & -1 & -3 & -3 \\ -1 & 3 & -1 & -2 \\ 1 & 3 & 2 & -2 \\ -3 & -2 & -3 & 0 \\ -2 & -2 & -2 & -3 \end{bmatrix}.$$

The value of entry $(2,4)$ in AB, i.e. $(AB)_{24}$, is:

☐ 2 ☐ -12 ☐ 21 ☐ -22 ☐ -13/12.

Problem 16 (5%)

The following commands are entered in the MATLAB Command Window:

```
>> A = [1 2 3 4; 1 3 5 7; 2 4 6 8]';   % bemærk apostrof
>> v = [1; 2; 3; 4];
>> T = [A v]
```

What is the result of the final command?

- ☐ `T` is a 3×4 matrix
- ☐ `T` is a 3×5 matrix
- ☐ `T` is a 4×4 matrix
- ☐ `T` is a 5×3 matrix
- ☐ MATLAB returns the error message: "Dimensions of matrices being concatenated are not consistent"