Exam in Linear Algebra

First Year at The Faculty of IT and Design and at the Faculty of Engineering and Science

23 August 2019, 9:00 – 13:00

This test consists of 11 pages and 12 problems. All problems are "multiple choice" problems. For each problem there is given a number of points; the subquestions are weighted uniformly within each problem.
The total for all 12 problems is 100 points.

It is allowed to use books, notes, xerox copies etc.
It is **not allowed** to use **any electronic devices**.

Your answers must be given in moodle by marking the relevant boxes.
The evaluation is only based on these markings.

In problems 10, 11 and 12 the evaluation is done following this principle:
Each wrong mark will annul one correct mark.

NAME:____________________________________

STUDENT NUMBER:____________________________________

Problem 1 (8 point)

This problem concerns the system of equations

$$\begin{array}{rcrcrcr} x_1 & - & 3x_2 & + & 2x_3 & = & -4 \\ -3x_1 & + & 9x_2 & - & 5x_3 & = & 14 \\ 2x_1 & - & 6x_2 & + & 4x_3 & = & -7 \end{array}.$$

1. Indicate which of the below matrices, that corresponds to the system's augmented coefficient matrix/total matrix $[A\ \mathbf{b}]$?

☐ $\begin{bmatrix} 1 & -3 & 2 \\ -3 & 9 & -6 \\ 2 & -5 & 4 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 & -3 & 2 \\ -3 & 9 & -5 \\ 2 & -6 & 4 \end{bmatrix}$

☐ $\begin{bmatrix} 1 & -3 & 2 \\ -3 & 9 & -6 \\ 2 & -5 & 4 \\ -4 & 14 & -7 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 & -3 & 2 & -4 \\ -3 & 9 & -5 & 14 \\ 2 & -6 & 4 & -7 \end{bmatrix}$

2. Which of the following matrices is the reduced row echelon matrix that is row equivalent to $[A\ \mathbf{b}]$?

☐ $\begin{bmatrix} 1 & -3 & 2 & -4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

☐ $\begin{bmatrix} 1 & -3 & 2 & -4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 & -3 & 0 & -8 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

3. Which of the following statements is true?

☐ The systemet is inconsistent.

☐ $x_1 = -2, x_2 = 2, x_3 = 2$ is one among several of the system's solutions.

☐ $x_1 = -3, x_2 = 2, x_3 = 2$ is the only solution to the system.

☐ $x_1 = 1, x_2 = 3, x_3 = 2$ is the only solution to the system.

☐ $x_1 = 1, x_2 = 3, x_3 = 2$ is one among several of the system's solutions.

☐ $x_1 = -8, x_2 = 0, x_3 = 2$ is the only solution to the system.

4. Indicate the true ones among the following statements:

☐ The vector $\mathbf{w} = (-8, 0, 2)$ is not a solution to the system.

☐ The vector $\mathbf{w} = (-8, 0, 2)$ fulfils $\mathbf{w} = A^{-1}\mathbf{b}$ for $\mathbf{b} = (-4, 14, -7)$.

☐ The vector $\mathbf{w} = (-8, 0, 2)$ solves $A\mathbf{x} = \mathbf{b}$, but it is not equal to $A^{-1}\mathbf{b}$.

☐ The coefficientmatrix A is not invertible.

Problem 2 (8 point)

This concerns the matrix $A = \begin{bmatrix} 1 & 2 & 4 & -6 & 4 \\ -2 & -4 & -7 & 11 & -7 \\ 2 & 4 & 6 & -9 & 8 \end{bmatrix}$, whereby the columns are denoted by $\mathbf{a}_j$ so that $A = [\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3\ \mathbf{a}_4\ \mathbf{a}_5]$.

1. Mark that or those systems of vectors that constitute(s) a basis for A's null space, Null A.

   □ $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ □ $\begin{bmatrix} 2 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ 0 \\ 6 \\ 4 \\ -2 \end{bmatrix}$

   □ $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ -3 \\ -2 \\ 1 \end{bmatrix}$ □ $\begin{bmatrix} 2 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ -3 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

2. Indicate the dimension of A's null space, dim Null A.

   □ 0 □ 1 □ 2 □ 3 □ 4 □ 5

3. Indicate the dimension of A's column space, dim Col A.

   □ 0 □ 1 □ 2 □ 3 □ 4 □ 5

4. The pivot columns in A constitute a basis $\mathcal{B}$ for Col A. Indicate which of the below vectors that is equal to $[\mathbf{a}_5]_{\mathcal{B}}$, i.e. the coordinate column of $\mathbf{a}_5$ with respect to $\mathcal{B}$.

   □ $\begin{bmatrix} 4 \\ -7 \\ 8 \end{bmatrix}$ □ $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ □ $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}$ □ $\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ □ $\begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$ □ $\begin{bmatrix} 4 \\ 0 \\ 3 \\ 2 \end{bmatrix}$

Problem 3 (8 point)

This concerns the matrices A and B and the vector $\mathbf{b}$ that are given by

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -3 \\ -3 & -8 & 4 \end{bmatrix}, \qquad B = \begin{bmatrix} 3 & 4 \\ 1 & 2 \\ 0 & 1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}.$$

1. Which of the following vectors coincides with the product $A\mathbf{b}$?

   □ $\begin{bmatrix} 8 \\ 2 \\ 0 \end{bmatrix}$ □ $\begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$ □ $\begin{bmatrix} 4 \\ 1 \\ -14 \end{bmatrix}$ □ $\begin{bmatrix} 6 \\ -3 \\ -7 \end{bmatrix}$ □ none of them

2. Which of the following matrices coincides with the product AB ?

   □ $\begin{bmatrix} 6 & 8 \\ 0 & 8 \\ 9 & 8 \end{bmatrix}$ □ $\begin{bmatrix} 5 & 8 \\ 1 & -1 \\ 9 & 8 \end{bmatrix}$ □ $\begin{bmatrix} 5 & 8 \\ 1 & -1 \\ -17 & -22 \end{bmatrix}$ □ $\begin{bmatrix} 5 & 9 \\ 1 & -1 \\ -17 & -24 \end{bmatrix}$

3. Which of the following matrices coincides with the inverse A^{-1} ?

   □ $\begin{bmatrix} -1 & 2 & -1 \\ 0 & -1 & 3 \\ 3 & 8 & -4 \end{bmatrix}$ □ $\begin{bmatrix} -1 & -4 & 7 \\ 2 & -1 & 2 \\ -1 & 2 & -3 \end{bmatrix}$ □ $\begin{bmatrix} 6 & 4 & 0 \\ -4 & 6 & 1 \\ 3 & 2 & 0 \end{bmatrix}$

   □ $\begin{bmatrix} -20 & -16 & -7 \\ 9 & 7 & 3 \\ 3 & 2 & 1 \end{bmatrix}$ □ $\begin{bmatrix} -20 & -16 & -7 \\ 9 & 6 & 1 \\ 3 & 2 & 4 \end{bmatrix}$ □ none of them

4. Which of the following matrices coincides with the product $A^{-1}B$?

   □ $\begin{bmatrix} -76 & -117 \\ -16 & -13 \\ 24 & 22 \end{bmatrix}$ □ $\begin{bmatrix} -76 & -119 \\ 34 & -14 \\ 23 & 22 \end{bmatrix}$ □ $\begin{bmatrix} -76 & -119 \\ 34 & 53 \\ 11 & 17 \end{bmatrix}$

   □ $\begin{bmatrix} -72 & -119 \\ 34 & -14 \\ 23 & 21 \end{bmatrix}$ □ $\begin{bmatrix} -76 & -119 \\ -34 & 53 \\ 23 & 22 \end{bmatrix}$ □ none of them

Problem 4 (8 point)

1. Mark the value of the determinant of the matrix $A = \begin{bmatrix} 0 & 0 & 3 \\ 3 & 2 & -1 \\ 5 & 0 & 1 \end{bmatrix}$.

   ☐ -13 ☐ -21 ☐ -30 ☐ 30 ☐ 21 ☐ 13

2. Mark the value of the determinant of the matrix $B = \begin{bmatrix} 0 & 1 & 0 \\ 4 & 0 & 2 \\ 0 & 3 & 0 \end{bmatrix}$.

   ☐ -10 ☐ -7 ☐ -1 ☐ 0 ☐ 1 ☐ 7 ☐ 10

3. Mark the value of the determinant of the matrix $C = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 5 & 25 \end{bmatrix}$.

   ☐ 0 ☐ 1 ☐ 6 ☐ 8 ☐ 10 ☐ 12

4. Which of the following numbers equals the determinant of $D = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 9 & 16 & 25 \end{bmatrix}$?

   ☐ 0 ☐ 2 ☐ 6 ☐ 8 ☐ 10 ☐ 12

Problem 5 (8 point)

The vectors $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \end{bmatrix}$, $\mathbf{u}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix}$ and $\mathbf{u}_4 = \begin{bmatrix} 0 \\ -2 \\ 0 \\ 1 \end{bmatrix}$ constitute a basis $\mathcal{B}$ for $\mathcal{R}^4$. By applying the Gram-Schmidt process to $\mathcal{B}$, one obtains an orthogonal basis consisting of the vectors $\mathbf{v}_1, \mathbf{v}_2$, $\mathbf{v}_3$ and $\mathbf{v}_4$. It holds true that:

1. ☐ $\mathbf{v}_1 = \frac{1}{\sqrt{2}}\mathbf{u}_1$ ☐ $\mathbf{v}_1 = \mathbf{u}_1$ ☐ none of them

2. ☐ $\mathbf{v}_2 = \mathbf{u}_2$ ☐ $\mathbf{v}_2 = \mathbf{u}_1 + \mathbf{u}_2$
   ☐ $\mathbf{v}_2 = \mathbf{u}_2 - \mathbf{u}_1$ ☐ none of them

3. ☐ $\mathbf{v}_3 = \mathbf{u}_3$ ☐ $\mathbf{v}_3 = \mathbf{u}_3 - \mathbf{u}_1 - 3\mathbf{v}_2$ ☐ $\mathbf{v}_3 = \mathbf{u}_3 - 2\mathbf{v}_1 - \mathbf{v}_2$
   ☐ $\mathbf{v}_3 = \mathbf{u}_3 - \mathbf{u}_1 - \mathbf{v}_2$ ☐ $\mathbf{v}_3 = \mathbf{u}_3 + 2\mathbf{u}_1 - 3\mathbf{v}_2$ ☐ none of them

4. ☐ $\mathbf{v}_4 = \mathbf{u}_4$ ☐ $\mathbf{v}_4 = \mathbf{u}_4 + \mathbf{u}_1 + \mathbf{v}_2 + \mathbf{v}_3$
   ☐ $\mathbf{v}_4 = \mathbf{u}_4 + \mathbf{u}_1$ ☐ $\mathbf{v}_4 = \mathbf{u}_4 + 2\mathbf{u}_1 - 3\mathbf{v}_2 + 4\mathbf{v}_3$
   ☐ $\mathbf{v}_4 = \mathbf{u}_4 - \mathbf{u}_1 - \mathbf{v}_2 - \mathbf{v}_3$ ☐ none of them

Problem 6 (8 point)

This concerns the matrix $A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 5 \end{bmatrix}$.

1. Which of the following polynomials is A's characteristic polynomium?

   □ $-\lambda^3 + 10\lambda^2 - 24\lambda$ □ $-\lambda^3 + 11\lambda^2 - 34\lambda + 24$

   □ $\lambda^3 - 10\lambda^2 + 25\lambda - 10$ □ none of them

2. Which of the following vectors are eigenvectors of A?

   □ $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ □ $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ □ $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ □ $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ □ $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ □ $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

3. Does the space $\mathcal{R}^3$ have a basis consisting of eigenvectors of A?

   □ Yes □ No

4. Which of the following numbers is equal to $\det(A)$?

   □ 8 □ 12 □ 23 □ 24 □ 25 □ 26

Problem 7 (8 point)

In the space $\mathcal{R}^3$ there is given a plane W by the equation $x_1 - 2x_2 - 2x_3 = 0$.

1. Mark the matrix which equals the ortogonal projection matrix P_W:

   □ $\frac{1}{9}\begin{bmatrix} 8 & 2 & 2 \\ 2 & 5 & -4 \\ 2 & -4 & 5 \end{bmatrix}$ □ $\frac{1}{9}\begin{bmatrix} -8 & -2 & -2 \\ -2 & -5 & 4 \\ -2 & 4 & -5 \end{bmatrix}$ □ $\frac{1}{3}\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

2. Mark the vector $\mathbf{w}$ in W, which is closest to the vector $\mathbf{v} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$:

   □ $\begin{bmatrix} \frac{-16}{9} \\ \frac{16}{9} \\ \frac{-24}{9} \end{bmatrix}$ □ $\begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$ □ $\begin{bmatrix} \frac{10}{9} \\ \frac{-20}{9} \\ \frac{25}{9} \end{bmatrix}$ □ $\begin{bmatrix} 3 \\ 1 \\ 8 \end{bmatrix}$ □ $\begin{bmatrix} \frac{8}{3} \\ 1 \\ \frac{3}{3} \end{bmatrix}$ □ none of them

Problem 8 (8 point)

A linear transformation $T : \mathbf{R}^n \to \mathbf{R}^m$ has the standard matrix

$$A = \begin{bmatrix} 1 & -1 & 2 & 2 & 4 \\ -1 & 1 & -2 & -1 & -1 \\ 3 & -3 & 6 & 4 & 6 \end{bmatrix}.$$

1. What is the value of the number n?

   ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5

2. What is the value of the number m?

   ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5

3. What is the rank of A?

   ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5

4. What is the dimension of the null space of A?

   ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5

5. What is the dimension of the column space of A?

   ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5

6. What is the dimension of the null space of the transposed matrix A^{T}, i.e. $\dim(\mathrm{Null}\, A^{\mathrm{T}})$?

   ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5

7. What is the dimension of the orthogonal complement to the null space of A, i.e. $\dim(\mathrm{Null}\, A)^{\perp}$?

   ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5

8. What is the dimension of the row space of A, i.e. $\dim(\mathrm{Row}\, A)$?

   ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5

Problem 9 (8 point)

This problem concerns the following system of ordinary differential equations:

$$y_1'(t) = 8y_1(t) - 6y_2(t), \qquad y_2'(t) = 9y_1(t) - 7y_2(t).$$

1. Mark the matrix that corresponds to the system's coefficient matrix A:

   □ $\begin{bmatrix} 1 & 1 \\ 8 & 9 \\ -6 & -7 \end{bmatrix}$ □ $\begin{bmatrix} 1 & 8 & -6 \\ 1 & 9 & -7 \end{bmatrix}$ □ $\begin{bmatrix} 8 & 9 \\ -6 & -7 \end{bmatrix}$ □ $\begin{bmatrix} 8 & -6 \\ 9 & -7 \end{bmatrix}$

2. Mark the set of eigenvalues of A:

   □ $\{-1, 2\}$ □ $\{1, -3\}$ □ $\{-2, 3\}$ □ $\{2, 3\}$ □ $\{2, -3\}$

3. Mark the system of vectors, which consists of eigenvectors of A:

   □ $\begin{bmatrix} -3 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ □ $\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ □ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ □ $\begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

4. Indicate the general solution to the system of differential equations, when a and b denote arbitrary real numbers:

   □ $y_1(t) = -3ae^{t} - be^{-3t}$,
   $y_2(t) = ae^{t} + be^{-3t}$.

   □ $y_1(t) = 2ae^{-3t} + be^{2t}$,
   $y_2(t) = 3ae^{-3t} + be^{2t}$.

   □ $y_1(t) = ae^{2t} + 2be^{3t}$,
   $y_2(t) = ae^{2t} + 3be^{3t}$.

   □ $y_1(t) = ae^{2t} + 2be^{-t}$,
   $y_2(t) = ae^{2t} + 3be^{-t}$.

Problem 10 (8 point, with annulment)

In this problem one considers the following 4 vectors:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ 4 \\ -1 \\ 0 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ 6 \\ 1 \\ -2 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} -1 \\ 2 \\ 5 \\ -6 \end{bmatrix}.$$

Is it true that

1. $\mathbf{v}_2$ is spanned by $\mathbf{v}_1$?

   ☐ Yes ☐ No

2. $\mathbf{v}_2$ is a linear combination of $\mathbf{v}_1$?

   ☐ Yes ☐ No

3. $\mathbf{v}_3$ is a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$?

   ☐ Yes ☐ No

4. $\mathbf{v}_4$ is a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$?

   ☐ Yes ☐ No

5. three of the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ and $\mathbf{v}_4$ generate the remaining one ?

   ☐ Yes ☐ No

6. the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ and $\mathbf{v}_4$ are linearly independent ?

   ☐ Yes ☐ No

7. the linear operator T on $\mathcal{R}^4$ having the standard matrix $A = [\mathbf{v}_1 \mathbf{v}_2 \mathbf{v}_3 \mathbf{v}_4]$ fulfils that $\dim \text{Null}\, T = 3$?

   ☐ Yes ☐ No

8. if a vector $\mathbf{w}$ in $\mathcal{R}^4$ belongs to the orthogonal complement $\text{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}^{\perp}$, then $\mathbf{w} = 0$?

   ☐ Yes ☐ No

Problem 11 (8 point, with annulment)

This problem concerns the following 4 matrices:

$$A = \begin{bmatrix} 5 & 2 \\ 0 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}, \; C = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}, \; D = \begin{bmatrix} -\frac{1}{2} & 1 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}.$$

1. Mark that or those matrices that are diagonalisable:

   ☐ A ☐ B ☐ C ☐ D

2. Mark that of those matrices, which A is similar to:

   ☐ $\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$ ☐ $\begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ☐ none of the matrices

3. Mark that or those matrices, which B is similar to:

   ☐ $\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$ ☐ $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ ☐ $\begin{bmatrix} 5 & 3 \\ 0 & 1 \end{bmatrix}$ ☐ none of the matrices

4. Mark that or those matrices, which C is similar to:

   ☐ $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ ☐ $\begin{bmatrix} -2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ ☐ $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ ☐ none of the matrices

5. Mark that or those matrices, which D is similar to:

   ☐ $\begin{bmatrix} -2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ☐ $\begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$ ☐ $\begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 7 & 0 & \frac{1}{2} \end{bmatrix}$ ☐ none of the matrices

6. Mark that or those matrices, which have 0 (zero) as an eigenvalue:

   ☐ A ☐ C ☐ none of them
   ☐ B ☐ D

7. Mark that or those matrices, which are invertible/regular:

   ☐ A ☐ C ☐ none of them
   ☐ B ☐ D

8. Mark that or those matrices that are orthogonal:

   ☐ A ☐ C ☐ none of them
   ☐ B ☐ D

Problem 12 (12 point, with annulment)

Mark every matrix, which has its eigenvalues among the numbers 1, 4 and 9:

- ☐ $\begin{bmatrix} 9 \end{bmatrix}$
- ☐ $\begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix}$
- ☐ $\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$
- ☐ $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$
- ☐ $\begin{bmatrix} 1 & 4 & 9 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$
- ☐ $\begin{bmatrix} 1 & 4 & 9 \\ 0 & 4 & 9 \\ 0 & 0 & 9 \end{bmatrix}$
- ☐ $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 4 & 0 \\ 9 & 0 & 0 \end{bmatrix}$
- ☐ $\begin{bmatrix} 9 & 0 & 0 \\ 0 & 0 & 4 \\ 1 & 0 & 0 \end{bmatrix}$
- ☐ $\begin{bmatrix} 9 & 0 & 0 \\ 0 & 4 & 2 \\ 0 & 2 & 5 \end{bmatrix}$
- ☐ $\begin{bmatrix} 9 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & -1 & 5 \end{bmatrix}$
- ☐ $\begin{bmatrix} 4 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 9 \end{bmatrix}$
- ☐ $\begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & -3 & -5 & 0 \\ 0 & 10 & 12 & 0 \\ 0 & 0 & 0 & 9 \end{bmatrix}$