Reexam in Linear Algebra

First Year at the Faculty of Engineering and Science
and the Technical Faculty of IT and Design

20 February 2019, 9:00-13:00

The present exam set consists of 8 numbered pages with 14 multiple-choice problems. A number of points are specified for each question. The total number of points equals 100. The exam is held as a digital exam.

Allowed aid: Books, notes, photocopies and prints.

Not allowed: Electronic aid such as calculator or mathematics program on the computer. Electronic documents.

See also the general guidelines for the exam.

Good luck!

Problem 1 (8 points)

Two matrices are given by

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}.$$

(a) (3 points). By matrix multiplication one gets the matrix $C = AB$. What is the entry c_{31}?

☐ 0	☐ 2	☐ -1
☐ 1	☐ -2	☐ 3

(b) (3 points). Let $D = A^{-1}$. What is the entry d_{31}?

☐ 0	☐ 2	☐ -1
☐ 1	☐ -2	☐ 3

(c) (2 points). Finally, one get the matrix $E = DC$ by matrix multiplication. What is the entry e_{12}?

☐ -3	☐ 0	☐ 4
☐ -2	☐ 1	☐ 5

Problem 2 (6 points)

A system of linear equations is given by

$$\begin{aligned} x_1 - x_2 + x_3 &= 6 \\ 2x_1 - 2x_2 + x_3 &= 4 \\ x_1 - x_2 - 2x_3 &= -18 \\ -x_1 + x_2 + x_3 &= 10. \end{aligned}$$

Mark the correct statement below.

☐ The only solution of the system is $x_1 = 1$, $x_2 = 3$, $x_3 = 8$.

☐ There are infinitely many solutions. One of these is $x_1 = 1$, $x_2 = 3$, $x_3 = 8$.

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☐ There are infinitely many solutions. One of these is $x_1 = 1$, $x_2 = 1$, $x_3 = 6$.

☐ The system is inconsistent.

☐ The system is consistent and it has no free variables.

Problem 3 (8 points)

A linear transformation $T : \mathcal{R}^3 \to \mathcal{R}^2$ is determined by

$$T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}3\\-1\end{bmatrix}, \quad T\left(\begin{bmatrix}1\\1\\0\end{bmatrix}\right) = \begin{bmatrix}2\\3\end{bmatrix}, \quad T\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right) = \begin{bmatrix}-1\\1\end{bmatrix}.$$

(a) (4 points). What is the standard matrix of T?

- ☐ $\begin{bmatrix}3 & -1\\2 & 3\\-1 & 1\end{bmatrix}$
- ☐ $\begin{bmatrix}3 & -1 & -3\\-1 & 4 & -2\end{bmatrix}$
- ☐ $\begin{bmatrix}1 & 5 & -1\\2 & 0 & -3\end{bmatrix}$
- ☐ $\begin{bmatrix}1 & 5\\-3 & 0\\2 & 3\end{bmatrix}$
- ☐ $\begin{bmatrix}3 & 2 & -1\\-1 & 3 & 1\end{bmatrix}$
- ☐ $\begin{bmatrix}1 & 0 & 0\\1 & 1 & 0\\1 & 1 & 1\end{bmatrix}$

(b) (2 points). Is T injective (one-to-one)?

- ☐ Yes
- ☐ No

(c) (2 points). Does there exist a vector $\mathbf{v} \neq \mathbf{0}$ such that $T(\mathbf{v}) = \mathbf{0}$?

- ☐ Yes
- ☐ No

Problem 4 (6 points)

A matrix is defined as

$$\begin{bmatrix}1 & 0 & 0\\-1 & 0 & 0\\5 & 3 & 2\end{bmatrix}.$$

(a) (3 points). What are the eigenvalues of the matrix?

- ☐ -2, 1 and 4
- ☐ -1 og 3
- ☐ 0, 1 and 2
- ☐ $\frac{5}{3}$, $\frac{3}{2}$ and 2
- ☐ 0 (with multiplicity 2) and 5
- ☐ there are none

(b) (3 points). Is the matrix diagonalizable?

☐ Yes ☐ No

Problem 5 (6 points)

A matrix is defined as

$$\begin{bmatrix} 1 & 0 & 5 & 1 \\ 2 & c & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 2 & 0 & 1 & -1 \end{bmatrix}$$

where c is a real constant.

(a) (3 points). What is the determinant of the matrix when $c = 0$?

☐ -16 ☐ -11 ☐ 3 ☐ 12

☐ -12 ☐ -8 ☐ 6 ☐ 13

(b) (3 points). For which value of c is the matrix not invertible?

☐ -10 ☐ -4 ☐ 0 ☐ 3

☐ -7 ☐ -1 ☐ 1 ☐ 4

Problem 6 (5 points)

A matrix Q is given by

$$Q = b \begin{bmatrix} 2 & 1 & a \\ -2 & a & 1 \\ 1 & 2 & -2 \end{bmatrix}$$

where a and b are real constants. Which combination makes Q an orthogonal matrix?

☐ $a = 2, b = \frac{1}{3}$

☐ $a = 1, b = \frac{1}{2}$

☐ $a = -1, b = \frac{1}{2}$

☐ $a = 0, b = \frac{1}{\sqrt{5}}$

☐ $a = 3, b = \frac{1}{\sqrt{14}}$

☐ $a = 2, b = \frac{1}{\sqrt{3}}$

Problem 7 (6 points)

Let A and B be two 4×4-matrices with determinants

$$\det(A) = 2, \quad \det(B) = -6.$$

Furthermore, let Q be an orthogonal 4×4-matrix.

(a) (2 points). What is $\det(-A)$?

☐ -12	☐ -6	☐ 2	☐ 10
☐ -10	☐ -2	☐ 6	☐ 12

(b) (2 points). What is $\det(A^{-1}B)$?

☐ -12	☐ -3	☐ 2	☐ 8
☐ -6	☐ -2	☐ 3	☐ 12

(c) (2 points). What is $\det(Q^2 A)$?

☐ -12	☐ -6	☐ 2	☐ 10
☐ -10	☐ -2	☐ 6	☐ 12

Problem 8 (6 points)

The linear transformation

$$T : \mathcal{R}^2 \to \mathcal{R}^2; \quad T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \frac{1}{13}\begin{bmatrix} 5 & 12 \\ 12 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

describes a reflection about a line through the origin. Which one of the following vectors is perpendicular to the line of reflection?

☐ $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ ☐ $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ ☐ $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$ ☐ $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ ☐ $\begin{bmatrix} -13 \\ 1 \end{bmatrix}$

Problem 9 (8 points)

A matrix and a vector are given by

$$A = \begin{bmatrix} -7 & -10 & 5 \\ 0 & 3 & 0 \\ -10 & -10 & 8 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

(a) (2 points). The vector **v** is an eigenvector of A. What is the corresponding eigenvalue?

☐ -7 ☐ -3 ☐ -2 ☐ 0 ☐ 5

(b) (3 points). It is given that $\lambda = 3$ is an eigenvalue of A. What is the dimension of the corresponding eigenspace?

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 7

(c) (3 points). Is A diagonalizable?

☐ Yes ☐ No

Problem 10 (10 points)

A plane W in $\mathcal{R}^3$ is determined by the equation $x_1 - 3x_2 + x_3 = 0$. Furthermore, a vector is given by

$$\mathbf{u} = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}.$$

(a) (7 points). Mark the orthogonal projection matrix P_W below.

☐ $\frac{1}{13}\begin{bmatrix} 11 & 4 & 1 \\ 4 & -2 & 3 \\ 1 & 3 & 5 \end{bmatrix}$ ☐ $\frac{1}{14}\begin{bmatrix} 13 & 2 & 3 \\ 2 & 10 & -6 \\ 3 & -6 & 5 \end{bmatrix}$ ☐ $\frac{1}{3}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

☐ $\frac{1}{11}\begin{bmatrix} 10 & 3 & -1 \\ 3 & 2 & 3 \\ -1 & 3 & 10 \end{bmatrix}$ ☐ $\frac{1}{6}\begin{bmatrix} 5 & 1 & 2 \\ 1 & 5 & -2 \\ 2 & -2 & 2 \end{bmatrix}$ ☐ $\frac{1}{2}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(b) (3 points). What is the distance from **u** to W?

☐ $\sqrt{14}$ ☐ $\sqrt{10}$ ☐ 8

☐ 5 ☐ $\sqrt{3}$ ☐ $\sqrt{11}$

Problem 11 (8 points)

Let $W = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ where

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}.$$

(a) (3 points). What is the dimension of W?

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

(b) (5 points). Which one of the following vectors does *not* lie in W?

☐ $\begin{bmatrix} 3 \\ -1 \\ 3 \\ 3 \end{bmatrix}$ ☐ $\begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{bmatrix}$ ☐ $\begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ ☐ $\begin{bmatrix} -1 \\ 0 \\ -1 \\ -1 \end{bmatrix}$ ☐ $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Problem 12 (10 points)

Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ be the basis for $\mathcal{R}^2$ with

$$\mathbf{b}_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

Furthermore, let $T : \mathcal{R}^2 \to \mathcal{R}^2$ be the linear transformation whose matrix representation with respect to $\mathcal{B}$ is

$$[T]_{\mathcal{B}} = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}.$$

(a) (7 points). What is the standard matrix A of T?

☐ $\begin{bmatrix} 1 & 4 \\ -1 & 11 \end{bmatrix}$ ☐ $\begin{bmatrix} 0 & 2 \\ -7 & 1 \end{bmatrix}$ ☐ $\begin{bmatrix} 3 & 9 \\ -1 & 0 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 & -2 \\ 2 & 10 \end{bmatrix}$

☐ $\begin{bmatrix} -4 & 2 \\ 9 & 1 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 & -4 \\ 4 & 2 \end{bmatrix}$ ☐ $\begin{bmatrix} -1 & -6 \\ 3 & 8 \end{bmatrix}$ ☐ $\begin{bmatrix} -3 & 1 \\ 12 & -5 \end{bmatrix}$

(b) (3 points). Mark the correct statement below.

☐ A is both diagonalizable and invertible.
☐ A has no eigenvalues.
☐ A is invertible but not diagonalizable.
☐ $\det(A) = 8$

Problem 13 (8 points)

A matrix is defined as

$$A = \begin{bmatrix} 1 & 1 & -4 & 2 \\ 2 & 0 & -6 & 9 \\ 2 & -1 & -5 & 7 \end{bmatrix}.$$

Mark a basis for the null space Null(A) below.

☐ $\left\{ \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

☐ $\left\{ \begin{bmatrix} 6 \\ 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \\ -1 \\ 0 \end{bmatrix} \right\}$

☐ $\left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ -1 \end{bmatrix} \right\}$

☐ $\left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 3 \end{bmatrix} \right\}$

☐ $\left\{ \begin{bmatrix} 3 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

☐ $\left\{ \begin{bmatrix} 4 \\ -4 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \end{bmatrix} \right\}$

Problem 14 (5 points)

In MATLAB's Command Window one types the following:

```
>> t = pi/5;
>> A = [ cos(t) -sin(t) ; sin(t) cos(t) ];
>> det(A)
```

What answer will MATLAB give?

☐
```
ans =
        0.6283
```

☐
```
ans =
        0.5000
```

☐
```
ans =
        2.0000
```

☐
```
ans =
        3.1416
```

☐
```
ans =
        1.0000
```

☐
```
ans =
        1.5708
```