Reexam in Linear Algebra

First Year at The Faculties of Engineering and Science and of Health

August 19, 2016, 9.00-13.00

This test has 11 pages and 14 problems. All the problems are "multiple choice" problems. **The answers must be given on these sheets.**

It is allowed to use books, notes, photocopies etc. It is **not** allowed to use any electronic devices such as pocket calculators, mobile phones or computers.

The listed percentages specify by which weight the individual problems influence the total examination.

Remember to write your full name (including middle names) together with your student number below.

NAME: _____

STUDENT NUMBER: _____

In all problems: *there is only one correct answer to each question.*

Problem 1 (4 %)

Let

$$A = \begin{bmatrix} 2 & 0 & 0 & 1 \\ 3 & 2 & 0 & 1 \\ 1 & 5 & 1 & 3 \\ -1 & 0 & 0 & 2 \end{bmatrix}.$$

What is the determinant of A?

☐ 12 ☒ 10 ☐ 5 ☐ 0 ☐ -5 ☐ -10 ☐ -12

Problem 2 (4 %)

Let A be a 4×6 matrix and let $E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$. How does the matrix EA from A?

☐ By transposing the matrix.
☐ By interchanging row 1 and row 3.
☒ By interchanging row 2 and row 4.
☐ By adding row 2 to row 4 and adding row 4 to row 2.

Problem 3 (10 %)

The matrix

$$A = \begin{bmatrix} 2 & 1 & -1 & 0 & 1 \\ 1 & -1 & 1 & -2 & 1 \\ 1 & 2 & -2 & 2 & 0 \\ 1 & 2 & -2 & 1 & 1 \end{bmatrix}$$

has reduced row echelon form

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

1. Which one of the following is a basis of the column space of A (Col A)?

□ $\left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -2 \\ -2 \end{bmatrix} \right\}$

☒ $\left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 2 \\ 1 \end{bmatrix} \right\}$

□ $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

□ $\left\{ \begin{bmatrix} -1 \\ 1 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

□ $\left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

2. Which one of the following is a basis of the row space of A (Row A)?

□ $\left\{ \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \\ 2 \\ 0 \end{bmatrix} \right\}$

□ $\left\{ \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \\ 1 \end{bmatrix} \right\}$

☒ $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} \right\}$

3. Which one of the following is a basis of the null space of A (Null A)?

□ $\left\{ \begin{bmatrix} -1 \\ 1 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

☒ $\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

□ $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

□ $\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

Problem 4 (10 %)

Let W be a subspace of $\mathbb{R}^4$ satifying that

$$W^{\perp} = \text{Span}\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} \right\}, \text{and let } \mathbf{u} = \begin{bmatrix} 8 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

1. Which one of the following is a basis of W

   ☐ $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ ☐ $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \right\}$ ☒ $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

2. Now let $\mathbf{w}$ be the orthogonal projection of $\mathbf{u}$ on W. What is the fourth component of $\mathbf{w}$ (i.e. w_4)

   ☐ -2 ☐ -1 ☐ 0 ☐ 1 ☒ 2 ☐ 4

3. Let $\mathbf{z}$ be the orthogonal projection of $\mathbf{u}$ on $W^{\perp}$. What is the fourth component of $\mathbf{z}$ (dvs. z_4)?

   ☒ -2 ☐ -1 ☐ 0 ☐ 1 ☐ 2 ☐ 4

Problem 5 (4 %)

Let $A = [\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3]$ be a matrix with 2 rows and let $B = [\mathbf{b}_1\ \mathbf{b}_2\ \mathbf{b}_3\ \mathbf{b}_4]$ be a matrix satisfying that the product $C = AB$ is defined.

1. How many rows are there in the matrix B?

   ☐ 2
   ☒ 3
   ☐ 4
   ☐ The number of rows can not be determined from the given information.

2. How many rows are there in the matrix C?

   ☒ 2
   ☐ 3
   ☐ 4
   ☐ The number of rows can not be determined from the given information.

Problem 6 (10 %)

The characteristic polynomial of

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 2 \end{bmatrix}$$

is $-(t+1)(t^2-6t+2)$.

1. Which one of the following is an eigenvalue of A?

   ☒ -1 ☐ 1 ☐ 2 ☐ 6

2. Which one of the following is an eigenvector of A?

   ☐ $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ ☐ $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$ ☒ $\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

3. Is A diagonalizable?

   ☒ Yes ☐ No

4. Is A invertible?

   ☒ Yes ☐ No

Problem 7 (8 %)

Let $A = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & 2 \\ 2 & -2 & 1 \end{bmatrix}$.

1. Let a be a positive number such that $Q = aA$ is an orthogonal matrix. What is the value of a?

   ☐ 1 ☐ 2 ☐ $\frac{1}{2}$ ☐ 3 ☒ $\frac{1}{3}$ ☐ $\frac{1}{9}$ ☐ 9

2. Let c be a number such that $A^{-1} = cA^{\mathrm{T}}$. What is the value of c?

   ☐ 1 ☐ 2 ☐ $\frac{1}{2}$ ☐ 3 ☐ $\frac{1}{3}$ ☒ $\frac{1}{9}$ ☐ 9

Problem 8 (8 %)

Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 6 & 5 & -4 \\ 7 & 8 & -9 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -4 \\ 3 & 3 & -5 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 0 \\ 7 \\ 6 \end{bmatrix}$.

1. Is $\mathbf{v}$ contained in Col A?

   ☒ Yes ☐ No

2. Is $\mathbf{v}$ contained in Null B?

   ☐ Yes ☒ No

3. Let $C = AB$. What is the value of c_{22}?

   ☐ -6 ☐ -4 ☐ -2 ☐ -1 ☐ 2 ☒ 4

Problem 9 (8 %)

1. ☐ What is the number of solutions of the following system of linear equations

$$\begin{aligned} x_1 \quad + 2x_3 + x_4 &= 1 \\ 2x_2 \quad + 2x_4 &= 2 \\ x_1 + 2x_2 + 2x_3 + 3x_4 &= 3 \end{aligned}$$

   ☐ 0
   ☐ 1
   ☐ 2
   ☒ infinitely many.

2. What is the nullity of the coefficient matrix?

   ☐ 0 ☐ 1 ☒ 2 ☐ 3 ☐ 4

3. What is the rank of the coefficient matrix?

   ☐ 0 ☐ 1 ☒ 2 ☐ 3 ☐ 4

Problem 10 (4 %)

Let A denote the matrix

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 13 & 17 & 19 \\ 9 & 29 & 31 & 37 \end{bmatrix},$$

and let B be a matrix in row echelon form, obtained from A by a number of elementary row operations. What is the value of b_{32}?

☐ -2
☒ 0
☐ 1
☐ 3
☐ 6
☐ This value can not be determined from the given information.

Problem 11 (5 %)

Let A be a non-zero 3×2 matrix.

1. What is the least possible rank of $A^{\mathrm{T}}A$?

   ☐ 0 ☒ 1 ☐ 2 ☐ 3

2. What is the largest possible rank of $A^{\mathrm{T}}A$?

   ☐ 0 ☐ 1 ☒ 2 ☐ 3

3. What is the largest possible rank of AA^{T}?

   ☐ 0 ☐ 1 ☒ 2 ☐ 3

Problem 12 (11 %)

Let

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 6 & 0 & 0 & 3 \end{bmatrix}$$

and let $B = A^{-1}$. Furthermore let C be a 4×4 matrix with $\det C = 4$.

1. What is the value of b_{41}?

   ☐ 0 ☐ 1 ☐ 2 ☐ $\frac{1}{2}$ ☒ -2 ☐ 6 ☐ $\frac{1}{6}$

2. What is the determinant of A?

   ☐ -2 ☐ 1 ☐ 6 ☒ 12 ☐ 14 ☐ 18 ☐ 72

3. What is the determinant of $A^{\mathrm{T}}C$?

☐ -8 ☐ 1 ☐ 3 ☐ 12 ☐ 16 ☒ 48 ☐ 72

4. What is the determinant of AC^{-1}?

☐ -8 ☐ 1 ☒ 3 ☐ 12 ☐ 16 ☐ 48 ☐ 72

5. What is the determinant of $-2C$?

☐ -64 ☐ -8 ☐ -4 ☐ -2 ☐ 2 ☐ 8 ☒ 64

Problem 13 (8 %)

Let $T : \mathbb{R}^m \to \mathbb{R}^n$ be a linear transformation with standard matrix

$$\begin{bmatrix} 2 & 3 & 1 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}.$$

1. What is the value of m?

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☒ 4 ☐ 5 ☐ 6

2. What is the value of n?

☐ 0 ☐ 1 ☐ 2 ☒ 3 ☐ 4 ☐ 5 ☐ 6

3. Is T onto?

☒ Yes ☐ No

4. What is the dimensionen of the null space of T?

☐ 0 ☒ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6

Problem 14 (6 %)

The following has been entered in the MATLAB Command Window:

```
>> A=[2 0 1; -1 2 1; 1 2 2; 1 0 0];
>> b=[2; 1; 3; 0];
>> rref([A b])

ans =

    1.0000         0         0         0
         0    1.0000         0   -0.5000
         0         0    1.0000    2.0000
         0         0         0         0

>> linsolve(A,b)
```

What is MATLABs answer to the last command?

☐ `Matrix is singular to working precision.`

☐
```
ans =

    2.0000
    1.0000
    3.0000
         0
```

☐
```
ans =

    0.0000
   -0.5000
    2.0000
         0
```

☒
```
ans =

    0.0000
   -0.5000
    2.0000
```