Exam in Linear Algebra

First Year at The Faculty of IT and Design and at the Faculty of Engineering and Science

14 June 2019, 9:00 – 13:00

This test consists of 11 pages and 12 problems. All problems are "multiple choice" problems. For each problem there is given a number of points; the subquestions are weighted uniformly within each problem.
The total for all 12 problems is 100 points.

It is allowed to use books, notes, xerox copies etc.
It is **not allowed** to use **any electronic devices**.

Your answers must be given in moodle by marking the relevant boxes.
The evaluation is only based on these markings.

In problems 10, 11 and 12 the evaluation is done following this principle:
Each wrong mark will annul one correct mark.

NAME:______________________________________

STUDENT NUMBER:______________________________________

Problem 1 (8 point)

This problem concerns the system of equations

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & x_3 & = & 3, \\ -2x_1 & + & 4x_2 & + & 3x_3 & = & -6, \\ 3x_1 & - & 6x_2 & - & 3x_3 & = & 9. \end{array}$$

1. Indicate which of the below matrices, that corresponds to the system's augmented coefficient matrix/total matrix $[A\ \mathbf{b}]$?

☐ $\begin{bmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ -1 & 3 & -3 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 & -2 & -1 \\ -2 & 4 & 3 \\ 3 & -6 & -3 \end{bmatrix}$

☐ $\begin{bmatrix} 1 & -2 & -1 & 3 \\ -2 & 4 & 3 & -6 \\ 3 & -6 & -3 & 9 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ -1 & 3 & -3 \\ 3 & -6 & 9 \end{bmatrix}$

2. Which of the following matrices is the reduced row echelon matrix that is row equivalent to $[A\ \mathbf{b}]$?

☐ $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

☐ $\begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 & -2 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

3. Which of the following statements is true?

☐ The systemet is inconsistent.

☐ $x_1 = 2, x_2 = 1, x_3 = 0$ is one among several of the system's solutions.

☐ $x_1 = 3, x_2 = 0, x_3 = 0$ is the only solution to the system.

☐ $x_1 = -1, x_2 = -2, x_3 = 0$ is the only solution to the system.

☑ $x_1 = -5, x_2 = -4, x_3 = 0$ is one among several of the system's solutions.

☐ $x_1 = -5, x_2 = -4, x_3 = 0$ is the only solution to the system.

4. Indicate the true ones among the following statements:

☐ The vector $\mathbf{w} = (-5, -4, 0)$ is not a solution to the system.

☐ The vector $\mathbf{w} = (-5, -4, 0)$ fulfils $\mathbf{w} = A^{-1}\mathbf{b}$ for $\mathbf{b} = (3, -6, 9)$.

☑ The vector $\mathbf{w} = (-5, -4, 0)$ solves $A\mathbf{x} = \mathbf{b}$, but it is not equal to $A^{-1}\mathbf{b}$.

☑ The coefficientmatrix A is not invertible.

Problem 2 (8 point)

This is about $A = \begin{bmatrix} 1 & -2 & 1 & 3 \\ 2 & -1 & 3 & 6 \\ -1 & 1 & -1 & -3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 9 \\ -2 \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} 1 \\ 1 \\ -2 \\ 1 \end{bmatrix}$, $\mathbf{d} = \begin{bmatrix} 6 \\ 0 \\ 0 \\ -2 \end{bmatrix}$.

1. Is $\mathbf{b}$ contained in the column space Col A?

   ☐ Yes ☐ No

2. Is $A \cdot \mathbf{c}$ contained in the column space Col A?

   ☐ Yes ☐ No

3. Is $\mathbf{c}$ contained in the null space Null A?

   ☐ Yes ☐ No

4. Is $\mathbf{d}$ contained in the null space Null A?

   ☐ Yes ☐ No

Problem 3 (8 point)

This concerns the matrices A and B and the vector $\mathbf{b}$ that are given by

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 2 \\ 2 & -2 & 3 \end{bmatrix}, \qquad B = \begin{bmatrix} 4 & 3 \\ 1 & 2 \\ 1 & 0 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}.$$

1. Which of the following vectors coincides with the product $A\mathbf{b}$?

   □ $\begin{bmatrix} 3 \\ -7 \\ 1 \end{bmatrix}$ □ $\begin{bmatrix} -7 \\ 2 \\ 11 \end{bmatrix}$ □ $\begin{bmatrix} 5 \\ 4 \\ -6 \end{bmatrix}$ □ $\begin{bmatrix} 7 \\ -2 \\ 12 \end{bmatrix}$ □ none of them

2. Which of the following matrices coincides with the product AB ?

   □ $\begin{bmatrix} 6 & 5 \\ 11 & 10 \\ 9 & 8 \end{bmatrix}$ □ $\begin{bmatrix} 3 & -1 \\ 5 & 6 \\ 9 & 2 \end{bmatrix}$ □ $\begin{bmatrix} 3 & 1 \\ 5 & 5 \\ 9 & 2 \end{bmatrix}$ □ $\begin{bmatrix} 6 & 4 \\ 5 & 8 \\ 11 & 5 \end{bmatrix}$ □ none of them

3. Which of the following matrices coincides with the inverse A^{-1} ?

   □ $\begin{bmatrix} -1 & 2 & -1 \\ 0 & -3 & -2 \\ -2 & 2 & -3 \end{bmatrix}$ □ $\begin{bmatrix} 6 & 1 & 0 \\ -2 & 6 & 1 \\ 3 & 2 & 4 \end{bmatrix}$ □ $\begin{bmatrix} -13 & -4 & 7 \\ -4 & -1 & 2 \\ 6 & 2 & -3 \end{bmatrix}$

   □ $\begin{bmatrix} -6 & -4 & 7 \\ -2 & -1 & 2 \\ 3 & 2 & -3 \end{bmatrix}$ □ $\begin{bmatrix} 6 & 4 & 0 \\ -2 & 6 & 1 \\ 3 & 2 & 0 \end{bmatrix}$ □ none of them

4. Which of the following matrices coincides with the product $A^{-1}B$?

   □ $\begin{bmatrix} -48 & -46 \\ -16 & -13 \\ 24 & 22 \end{bmatrix}$ □ $\begin{bmatrix} -49 & -47 \\ -15 & -14 \\ 23 & 22 \end{bmatrix}$ □ $\begin{bmatrix} -49 & -46 \\ -15 & -13 \\ 24 & 22 \end{bmatrix}$

   □ $\begin{bmatrix} -49 & -47 \\ -16 & -14 \\ 23 & 21 \end{bmatrix}$ □ $\begin{bmatrix} -48 & -46 \\ -15 & -13 \\ 23 & 22 \end{bmatrix}$ □ none of them

Problem 4 (8 point)

Here we look at 7×7-matrices A, B that fulfil $\det A = -4$ and $\det(AB) = 12$.

1. What is the value of $\det(\frac{1}{2}A)$?

   ☐ -128 ☐ -32 ☐ $-\frac{1}{32}$ ☐ $\frac{1}{32}$ ☐ 32 ☐ 128

2. What is the value of $\det B$?

   ☐ 16 ☐ 3 ☐ 2 ☐ -2 ☐ -3 ☐ -16

3. What is the value of $\det(A^{-1})$?

   ☐ $-\frac{1}{512}$ ☐ $-\frac{1}{128}$ ☐ $-\frac{1}{4}$ ☐ $\frac{1}{4}$ ☐ $\frac{1}{128}$ ☐ $\frac{1}{512}$

4. What is the value of $\det(B^T(A^{-1})^T)$?

   ☐ 12 ☐ $\frac{4}{3}$ ☐ $\frac{3}{4}$ ☐ $-\frac{3}{4}$ ☐ $-\frac{4}{3}$ ☐ -12

Problem 5 (8 point)

The vectors $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}$, $\mathbf{u}_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ og $\mathbf{u}_4 = \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}$ constitute a basis $\mathcal{B}$ for $\mathcal{R}^4$. By applying the Gram-Schmidt process to $\mathcal{B}$, one obtains an orthogonal basis consisting of the vectors $\mathbf{v}_1, \mathbf{v}_2$, $\mathbf{v}_3$ and $\mathbf{v}_4$. It holds true that:

1. ☐ $\mathbf{v}_1 = \mathbf{u}_1$ ☐ $\mathbf{v}_1 = \frac{1}{\sqrt{2}}\mathbf{u}_1$ ☐ none of the previous

2. ☐ $\mathbf{v}_2 = \mathbf{u}_2 - \mathbf{u}_1$ ☐ $\mathbf{v}_2 = \mathbf{u}_2 + \mathbf{u}_1$ ☐ none of the previous

3. ☐ $\mathbf{v}_3 = \mathbf{u}_3$ ☐ $\mathbf{v}_3 = \mathbf{u}_3 - \mathbf{u}_1 - 3\mathbf{v}_2$ ☐ $\mathbf{v}_3 = \mathbf{u}_3 - \mathbf{v}_1 - \mathbf{v}_2$
   ☐ $\mathbf{v}_3 = \mathbf{u}_3 + \mathbf{u}_1 - \mathbf{v}_2$ ☐ $\mathbf{v}_3 = \mathbf{u}_3 + 2\mathbf{u}_1 - 3\mathbf{v}_2$ ☐ none of the previous

4. ☐ $\mathbf{v}_4 = \mathbf{u}_4$ ☐ $\mathbf{v}_4 = \mathbf{u}_4 + \mathbf{u}_1 - \mathbf{v}_2 + \mathbf{v}_3$
   ☐ $\mathbf{v}_4 = \mathbf{u}_4 + \mathbf{u}_1$ ☐ $\mathbf{v}_4 = \mathbf{u}_4 + 2\mathbf{u}_1 - 3\mathbf{v}_2 + 4\mathbf{v}_3$
   ☐ $\mathbf{v}_4 = \mathbf{u}_4 - \mathbf{u}_1 - \mathbf{v}_2 - \mathbf{v}_3$ ☐ none of the previous

Problem 6 (8 point)

In this problem there are three vectors in $\mathcal{R}^3$ given by

$$\mathbf{a} = \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \qquad \mathbf{c} = \begin{bmatrix} 2 \\ -5 \\ t \end{bmatrix}$$

Notice that the last coordinate t in the vector $\mathbf{c}$ is a real variable.
Mark the true ones among the following statements:

1. $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are linearly dependent vectors for

   ☐ $t = 9$ ☐ $t = -5$ ☐ all real numbers t
   ☐ $t \neq -9$ ☐ $t \neq -5$

2. $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ span $\mathcal{R}^3$ for

   ☐ $t = -9$ ☐ $t = -5$ ☐ all real numbers t
   ☐ $t \neq 9$ ☐ $t \neq -5$

3. $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ constitue a basis $\mathcal{B}$ for $\mathcal{R}^3$ for

   ☐ $t = 9$ ☐ $t = -4$
   ☐ $t = 4$ ☐ all real numbers t

4. for $t = 4$ the vector $\mathbf{v} = (-1, 4, 2)$ has its $\mathcal{B}$-coordinate column $[\mathbf{v}]_{\mathcal{B}}$ given by

   ☐ $\begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 \\ -8 \\ 0 \end{bmatrix}$ ☐ $\begin{bmatrix} 0 \\ -5 \\ 1 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

Problem 7 (8 point)

In the space $\mathcal{R}^3$ there is given a plane W by the equation $x_1 + 1x_2 - 2x_3 = 0$.

1. Mark the matrix, which equals the orthogonal projection matrix P_W:

   ☐ $\frac{1}{12}\begin{bmatrix} 11 & -2 & 4 \\ -2 & 11 & 4 \\ 4 & 4 & -2 \end{bmatrix}$ ☐ $\frac{1}{6}\begin{bmatrix} 5 & 1 & 2 \\ 1 & 5 & -2 \\ 2 & -2 & 2 \end{bmatrix}$ ☐ $\frac{1}{6}\begin{bmatrix} 5 & -1 & 2 \\ -1 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix}$

2. Mark the orthogonal projection $P_W\mathbf{v}$ of $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ on the plane W:

   ☐ $\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ ☐ $\frac{1}{6}\begin{bmatrix} 11 \\ 3 \\ 6 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$ ☐ $\frac{1}{2}\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$

Problem 8 (8 point)

A linear transformation $T : \mathbf{R}^n \to \mathbf{R}^m$ has the standard matrix $A = \begin{bmatrix} 1 & -2 & 3 & -2 \\ 3 & -6 & 7 & -2 \\ -2 & 4 & -8 & 3 \end{bmatrix}$.

1. What is the value of the number n?

   ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5

2. What is the value of the number m?

   ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5

3. What is the rank of A?

   ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5

4. What is the dimension of the null space of A?

   ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

5. What is the dimension of the column space of A?

   ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

6. What is the dimension of the null space of the transposed matrix A^{T}, i.e. $\dim(\mathrm{Null}\, A^{\mathrm{T}})$?

   ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

7. What is the dimension of the orthogonal complement to the null space of A, i.e. $\dim(\mathrm{Null}\, A)^{\perp}$?

   ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

8. What is the dimension of the row space of A, i.e. $\dim(\mathrm{Row}\, A)$?

   ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

Problem 9 (8 point)

This problem concerns the following system of ordinary differential equations:

$$y_1'(t) = y_1(t) + 2y_2(t), \qquad y_2'(t) = -y_1(t) + 4y_2(t).$$

1. Mark the matrix that corresponds to the system's coefficient matrix A:

   ☐ $\begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ ☐ $\begin{bmatrix} 0 & 6 \\ -2 & 2 \end{bmatrix}$

2. Mark the set of eigenvalues of A:

   ☐ $\{1,2\}$ ☐ $\{1,3\}$ ☐ $\{2\}$ ☐ $\{2,3\}$ ☐ $\{2,-3\}$

3. Mark the system of vectors, which consists of eigenvectors of A:

   ☐ $\begin{bmatrix} 1 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ ☐ $\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ ☐ $\begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

4. Indicate the general solution to the system of differential equations, when a and b denote arbitrary real numbers:

   ☐ $y_1(t) = ae^{2t} + 2be^{3t}$, $y_2(t) = -ae^{2t} + 4be^{3t}$.

   ☐ $y_1(t) = 2ae^{2t} + 3be^{3t}$, $y_2(t) = 2ae^{2t} + 3be^{3t}$.

   ☐ $y_1(t) = 2ae^{2t} + be^{3t}$, $y_2(t) = ae^{2t} + be^{3t}$.

   ☐ $y_1(t) = 2ae^{2t} + be^{3t}$, $y_2(t) = -ae^{2t} + 4be^{3t}$.

Problem 10 (8 point, with annulment)

In this problem one considers the following 4 vectors:

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ 3 \\ 1 \\ -1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -3 \\ -4 \\ -1 \\ 2 \end{bmatrix} \mathbf{v}_3 = \begin{bmatrix} -2 \\ -2 \\ -1 \\ 1 \end{bmatrix} \mathbf{v}_4 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}.$$

Is it true that

1. $\mathbf{v}_2$ is not proportional to $\mathbf{v}_1$?

   ☐ Yes ☐ No

2. $\mathbf{v}_2$ is a linear combination of $\mathbf{v}_1$?

   ☐ Yes ☐ No

3. $\mathbf{v}_3$ is a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$?

   ☐ Yes ☐ No

4. $\mathbf{v}_4$ is a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$?

   ☐ Yes ☐ No

5. three of the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ and $\mathbf{v}_4$ generate the remaining one ?

   ☐ Yes ☐ No

6. the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ and $\mathbf{v}_4$ are linearly independent ?

   ☐ Yes ☐ No

7. Every vector $\mathbf{w}$ i $\mathcal{R}^4$ belongs to the subspace $\text{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$?

   ☐ Yes ☐ No

8. the linear operator T on $\mathcal{R}^4$ having the standard matrix $A = [\mathbf{v}_1\mathbf{v}_2\mathbf{v}_3\mathbf{v}_4]$ is surjective ?

   ☐ Yes ☐ No

Problem 11 (8 point, with annulment)

This problem concerns the following 4 matrices:

$$A = \begin{bmatrix} 5 & 3 \\ 0 & 1 \end{bmatrix}, \ B = \begin{bmatrix} 1 & -3 \\ 0 & 5 \end{bmatrix}, \ C = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}, \ D = \begin{bmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}.$$

Is it true that

1. A and B are inverses of one another?

   ☐ Yes ☐ No

2. C are D are inverses of one another?

   ☐ Yes ☐ No

3. A is invertible?

   ☐ Yes ☐ No

4. B is diagonalisable?

   ☐ Yes ☐ No

5. Mark that matrix, or those matrices, which B is similar to:

   ☐ $\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$ ☐ $\begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$ ☐ $\begin{bmatrix} 5 & 3 \\ 0 & 1 \end{bmatrix}$ ☐ none of the matrices

6. C is diagonalisable?

   ☐ Yes ☐ No

7. D is diagonalisable?

   ☐ Yes ☐ No

8. Mark that or those diagonal matrices, which D is similar to:

   ☐ $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$ ☐ $\begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$ ☐ none of the matrices

Problem 12 (12 point, with annulment)

Mark those among the below matrices, which have 2, 3 and 7 as eigenvalues:

☐ $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ ☐ $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ ☐ $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$

☐ $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ ☐ $\begin{bmatrix} 7 & 3 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{bmatrix}$ ☐ $\begin{bmatrix} 7 & 3 & 2 \\ 0 & 7 & 3 \\ 0 & 0 & 7 \end{bmatrix}$

☐ $\begin{bmatrix} 2 & 0 & 0 \\ 3 & 3 & 0 \\ 0 & 7 & 7 \end{bmatrix}$ ☐ $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 7 & 0 \end{bmatrix}$ ☐ $\begin{bmatrix} 0 & 0 & 3 \\ 0 & 7 & 0 \\ 2 & 0 & 0 \end{bmatrix}$

☐ $\begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & 10 \\ 0 & -5 & 12 \end{bmatrix}$ ☐ $\begin{bmatrix} 7 & 0 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & -1 & 5 & 0 \\ 0 & 0 & 0 & 7 \end{bmatrix}$ ☐ $\begin{bmatrix} 7 & 0 & 0 & 0 \\ 0 & 27 & -5 & 0 \\ 0 & 100 & -18 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$