FACIT

Examn in linear algebra, January 8., 2013

Ex. 1

1) $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

2) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

3) No solution

Ex. 2

1) $\begin{bmatrix} 20 \\ 50 \end{bmatrix}$ 2) not possible 3) $\begin{bmatrix} 9 & 12 & 15 \\ 19 & 26 & 33 \end{bmatrix}$

4) not possible

Ex 3

$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

Ex 4

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$

Ex 5

1) $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$T\left(\begin{bmatrix} -1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

2) $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

3) Reflection about the y-axis

EX 6 1) $\lambda_1 = 1$ (multiplicity 1)

$\lambda_2 = 0$ (multiplicity 2)

2) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$ and $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

$\uparrow$ $\nearrow$

other answers are possible

3) A is diagonalizable

because ...

4) Either note $A^2 = A$ and therefore $A^{1057} = A$

or employ

$$A^{1057} = P D^{1057} P^{-1}.$$

EX 7 1) check inner product and length ....

2) $\bar{\omega} = \frac{1}{5} \begin{bmatrix} 9 \\ 10 \\ 12 \end{bmatrix}$

$\bar{z} = \frac{1}{5} \begin{bmatrix} -4 \\ 0 \\ 3 \end{bmatrix}$

3) $\begin{pmatrix} 9/25 & 0 & 12/25 \\ 0 & 1 & 0 \\ 12/25 & 0 & 16/25 \end{pmatrix}$

4) One possible answer is

$$\left\{ \begin{bmatrix} -4/3 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Ex 8

1) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 4 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ 2 \\ 0 \end{bmatrix} \right\}$

2) Not orthonormal.

Part II ("multiple choice" exercises)

Exercise 9 (5%).

The following informations are given. A is a 3×3 symmetric matrix. The vector **u** is an eigenvector for A with eigenvalue equal to 2. Furthermore, the vector **v** is an eigenvector for A with eigenvalue equal to -1.
Exactly one of the following statements is correct. Tick off the correct answer.

- [x] $\mathbf{u} \cdot \mathbf{v} = 0$.
- [ ] $\mathbf{u} \cdot \mathbf{v} = 2$.
- [ ] $\mathbf{u} \cdot \mathbf{v} = -2$.

Exercise 10 (6%).

A is an $n \times n$ matrix such that $\det(A^3) = -27$. For each of the following two questions tick off the correct answer.

$\det(A) =$

- [ ] -24
- [ ] -9
- [x] -3
- [ ] 3
- [ ] 9
- [ ] 24

$\det(A^T A^{-1}) =$

- [ ] -81
- [ ] -9
- [ ] -1
- [x] 1
- [ ] 9
- [ ] 81

Exercise 11 (10%).

Consider

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \end{bmatrix}, \mathbf{u}_3 = \begin{bmatrix} 2 \\ 4 \\ 2 \\ 0 \end{bmatrix}, \mathbf{u}_4 = \begin{bmatrix} -1 \\ -2 \\ 0 \\ -3 \end{bmatrix}.$$

Let $A = [\mathbf{u}_1\, \mathbf{u}_2\, \mathbf{u}_3\, \mathbf{u}_4]$ and $W = \mathrm{Span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$.

Tick off correct statements among the ten statements below.

(Only the statements that you have ticked off will contribute to your mark. Among the statements you have ticked off every incorrect statement will neutralize a correct statement. So if you ticked off 5 statements of which 4 are correct and 1 is wrong, you will receive credits for 4-1=3 correct answers. If you ticked of 4 statements of which 2 are correct and 2 are incorrect, you will receive credits for 2-2=0 correct answer (= no credit). You cannot receive a negative number of points. Hence, if you ticked off 4 statements of which 1 is correct and 3 are incorrect you will receive credits for 0 correct answer (= no credit).)

☒ The dimension of W is 2.
☐ The dimension of W is 3.
☐ The dimension of W is 4.
☐ A is invertible (regular).
☒ $\det(A) = 0$.

☐ $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is a basis for W.
☒ $\{\mathbf{u}_1, \mathbf{u}_2\}$ is a basis for W.
☐ $\{\mathbf{u}_1, \mathbf{u}_3\}$ is a basis for W.
☒ $\{\mathbf{u}_1, \mathbf{u}_4\}$ is a basis for W.
☒ $\mathrm{Span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\} = W$.

Exercise 12 (8%).

Let $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}$ and define the function $T : \mathbb{R}^2 \to \mathbb{R}^3$ by $T(\mathbf{x}) = A\mathbf{x}$.

Answer the following 5 true/false exercises:

a. T is a linear transformation.

☒ True ☐ False

b. T is onto (surjective)

☐ True ☒ False

c. T is one-to-one (injective).

☒ True ☐ False

d. T is invertible.

☐ True ☒ False

e. T corresponds to a rotation.

☐ True ☒ False