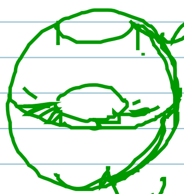
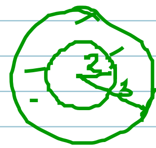
Volumen ved polær integration



$x^2+y^2+z^2=9$



$x^2+y^2=4$

$0 \le \theta \le 2\pi \qquad 2 \le r \le 3$

$z_{top}, z_{bund} \qquad z^2 = 9-x^2-y^2 = 9-(\underbrace{x^2+y^2}_{r^2})$

$9-r^2$

$z_{top}=\sqrt{9-r^2}, \; z_{bund} = -\sqrt{9-r^2}$

$$\int_0^{2\pi}\int_2^3 \left(\sqrt{9-r^2}+\sqrt{9-r^2}\right)\cdot r \, dr \, d\theta$$

$$= 2\cdot\int_0^{2\pi}\int_2^3 \sqrt{9-r^2}\cdot r \, dr \, d\theta$$

$$= 2\int_0^{2\pi}\left[-\frac{1}{3}(9-r^2)^{\frac{3}{2}}\right]_2^3 d\theta = \frac{20\sqrt{5}}{3}\cdot 11$$

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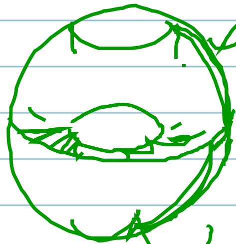
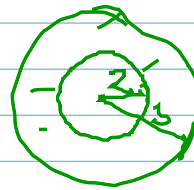
$x^2+y^2+z^2=9$



$x^2+y^2=4$



$$0 \leq \theta \leq 2\pi \qquad 2 \leq r \leq 3$$

z_{top}, z_{bund} $\qquad z^2 = 9 - x^2 - y^2 = 9 - (\underbrace{x^2+y^2}_{r^2})$

$$9 - r^2$$

$$z_{top} = \sqrt{9-r^2}, \quad z_{bund} = -\sqrt{9-r^2}$$

$$\int_0^{2\pi} \int_2^3 \left(\sqrt{9-r^2} + \sqrt{9-r^2}\right) \cdot r \, dr \, d\theta$$

$$= 2 \cdot \int_0^{2\pi} \int_2^3 \sqrt{9-r^2} \cdot r \, dr \, d\theta$$

$$= 2 \int_0^{2\pi} \left[-\frac{1}{3} (9-r^2)^{\frac{3}{2}} \right]_2^3 d\theta = \frac{20\sqrt{5}}{3} \cdot \pi$$