Test Exam in Calculus

Set 2 May 2011

First Year at The TEK-NAT Faculty and Health Faculty

The present exam consists of 7 numbered pages with a total of 12 exercises.

It is allowed to use books, notes, etc. It is **not** allowed to use electronic devices.

The listed percentages specify by which weight the individual exercises influence the total examination.

This exam set has two independent parts.

- Part I contains "regular exercises". Here it is important that you explain the idea behind the solution, and that you provide relevant intermediate results.
- Part II is "multiple choice" exercises. The answers for Part II must be given on these sheets

Remember to write your full name (including middle names), student number together with the course number on each side of your answers. **Number each page and write the total number of pages on the front page of the answers.**

Good luck!

NAME: ______________________________

STUDENT NUMBER: ______________________________

COURSE NUMBER: ______________________________

Part I (”regular exercises”)

Exercise 1 (10%)

Determine the Taylor polynomium of degree 3 for the function

$$f(x) = \arcsin(2x),$$

around $a = 0$.

Exercise 2 (9%)

A surface $\mathcal{F}$ is given implicitly as the solution of the equation

$$F(x, y, z) = x^4 + y^4 + z^4 + 2x^2y^2z^2 - 26 = 0.$$

(a) Determine the gradient vector $\nabla F(x, y, z)$.

(b) Determine an equation for the tangent plane to the surface $\mathcal{F}$ through the point $(1, -1, 2)$.

Exercise 3 (10%)

A thin plate covers exactly the area

$$R = \{(x, y) \in \mathbf{R}^2 : x^2 + y^2 \leq 4,\ y \geq 0\}$$

in the xy plane. The density of the plate is $\delta(x, y) = \sqrt{x^2 + y^2}$.

(a) Determine the mass of the plate.

(b) Let $(\bar{x}, \bar{y})$ denote the centroid of the plate. By symmetry $\bar{x} = 0$. Determine $\bar{y}$.

Exercise 4 (8%)

Consider the 4th degree equation

$$z^4 - (3 + i)z^2 + 3i = 0.$$

(a) How many complex roots does the equation have (counted with multiplicity)?

(b) Solve the equation (hint: let $w = z^2$).

Exercise 5 (5%)

Consider the function

$$f(x,y) = \arccos[4(x^2 + y^2)].$$

(a) Determine the domain of f.

(b) Determine the partial derivatives $f_x(x,y)$ and $f_y(x,y)$.

Exercise 6 (10%)

Consider the function

$$f(x,y) = x^2 - y^2$$

on the area

$$R = \{(x,y) \in \mathbf{R}^2 : x^2 + y^2 \leq 1\}.$$

(a) Determine the critical points for f in the inside of R.

(b) Find the extrema (minimum and maximum) for f on R.

Exercise 7 (12%)

(a) Find the uniquely determined solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0,$$

that fulfills

$$y(0) = 1 \quad \text{og} \quad y'(0) = 0.$$

(b) We are told that

$$y_1(x) = \frac{1}{2} + \frac{3}{50}\cos(2\,x) - \frac{2}{25}\sin(2\,x)$$

is a particular solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = \sin^2(x).$$

Find a particular solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{2x} - \sin^2(x).$$

Exercise 8 (10%)

A curve in the plane is given by

$$\begin{aligned} x(t) &= 3\sin(t) \\ y(t) &= 5\cos(t), \qquad t \in \mathbf{R}. \end{aligned}$$

Compute the curvature of the curve $\kappa(t)$.

Part II (”multiple choice” exercises)

Exercise 9 (7%)

Consider the triple integral

$$I := \iiint_T (x^2y + xz)\, dV,$$

where $T = [0,1] \times [-2,3] \times [-1,1]$. Which of the 4 iterated integrals below can be used to determine the mass of I. (note: the value of I should *not* be calculated!)

□ $\int_{-2}^{3} \int_{-1}^{1} \int_{0}^{1} (x^2y + xz) dx\, dy\, dz.$

□ $\int_{0}^{1} \int_{-2}^{3} \int_{-1}^{1} (x^2y + xz) dx\, dz\, dy.$

□ $\int_{0}^{1} \int_{-1}^{1} \int_{-2}^{3} (x^2y + xz) dy\, dz\, dx.$

□ $\int_{0}^{1} \int_{-2}^{3} \int_{-1}^{1} (xy^2 + xz) dz\, dy\, dx.$

Exercise 10 (6%)

Consider a complex polynomial $p(z)$ of degree 7. Mark all correct statements below.

□ $p(z)$ always has at least one real root
□ $p(z)$ has exactly 7 different complex roots
□ $p(z)$ has exactly 7 complex roots counted with multiplicity
□ $p(z)$ has at least to different roots.

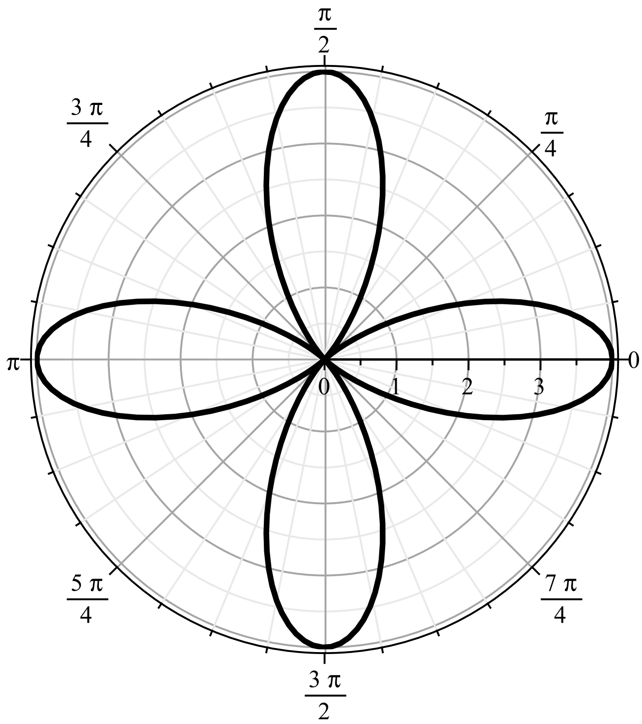
Exercise 11 (6%)

Consider a function $f(x,y)$ of two variables defined on $\mathbf{R}^2$. The function has continuous partial derivatives for all $(x,y) \in \mathbf{R}^2$. A point $P(a,b)$ i $\mathbf{R}^2$ is given where we are told that $f_x(a,b) \neq 0$. Mark all correct statements below.

- [ ] f is increasing in $P(a,b)$ in the direction of the x axis
- [ ] There exists a unit vector $\mathbf{u}$, such that f is increasing in $P(a,b)$ in the direction given by $\mathbf{u}$
- [ ] f is increasing in $P(a,b)$ in the direction of the y axis.

Exercise 12 (6%)

The figure below shows the graph of the function $r = f(\theta)$ in polar coordinates.



Which of the below expressions for f along with the domain for the θ gives the above figure.

- ☐ $f(\theta) = 1 + \cos(4\theta),\ 0 \leq \theta \leq 2\pi$
- ☐ $f(\theta) = 2 + 2\cos(2\theta),\ 0 \leq \theta \leq \pi$
- ☐ $f(\theta) = 2 + 2\sin(4\theta),\ 0 \leq \theta \leq 2\pi$
- ☐ $f(\theta) = 2 + 2\cos(4\theta),\ 0 \leq \theta \leq 2\pi$
- ☐ $f(\theta) = 2 + \cos(4\theta),\ 0 \leq \theta \leq \pi$
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