Test Exam in Calculus

Set 1 April 2011

First Year at The TEK-NAT Faculty and Health Faculty

The present exam consists of 7 numbered pages with a total of 12 exercises.

It is allowed to use books, notes, etc. It is **not** allowed to use electronic devices.

The listed percentages specify by which weight the individual exercises influence the total examination.

This exam set has two independent parts.

- Part I contains "regular exercises". Here it is important that you explain the idea behind the solution, and that you provide relevant intermediate results.
- Part II is "multiple choice" exercises. The answers for Part II must be given on these sheets

Remember to write your full name (including middle names), student number together with the course number on each side of your answers. **Number each page and write the total number of pages on the front page of the answers.**

Good luck!

NAME: ______________________________

STUDENT NUMBER: ______________________________

COURSE NUMBER: ______________________________

Part I ("regular exercises")

Exercise 1 (12%)

(a) Find the uniquely determined solution to the differential equation

$$y'' - 2y' + 5y = 0,$$

that fulfills

$$y(0) = 0 \quad \text{og} \quad y'(0) = 2.$$

(b) Find the complete solution to the differential equation

$$y'' - 2y' + 5y = 2x + 3.$$

Exercise 2 (10%)

A function $f(x)$ is given with the property that its Taylor polynomial of degree 3 around $a = 0$ is

$$P_3(x) = 6 + 2x - x^2 + 2x^3.$$

(a) Determine the value $f(0)$

(b) Determine the value $f'(0)$

(c) Determine the value $f''(0)$

(d) Determine the value $f'''(0)$.

Exercise 3 (7%)

Consider

$$f(x,y) = 9 + y - y^2 + 2x + xy - 2xy^2.$$

(a) Determine the partial derivative $\frac{\partial f}{\partial x}(x,y)$

(b) Determine the partial derivative $\frac{\partial f}{\partial y}(x,y)$

(c) Determine an equation for the tangent plane to the surface $z = f(x,y)$ through the point $(-1,1,8)$.

Exercise 4 (10%)

Compute the double integral

$$\iint_R x^2 \, dA,$$

where R is bounded in the xy plane by the curves $y = x^2$ and $y = x^3$.

Exercise 5 (7%)

Solve the 2. degree equation

$$z^2 - (1+i)z + 2 + 2i = 0.$$

Exercise 6 (8%)

Consider the surface (ellipsoid) given implicitly as the solution of the equation

$$x^2 + 3y^2 + 4z^2 = 20.$$

Find the tangent plane to the surface through the point $P(1, 1, 2)$.

Exercise 7 (11%)

Consider the function

$$f(x, y, z) = e^{x+y} + 2z^2.$$

(a) Determine the gradient vector $\nabla f(x, y, z)$.

(b) Determine the directional derivative f in the point $P(0, 0, 1)$ in the direction given by $\mathbf{v} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

Exercise 8 (10%)

The body T in space is bounded by the surfaces $z = x^2 + y^2$ and $z = 1$. The density of T is given by $\delta(x, y, z) = z^2$.

(a) Determine the mass of T.

(b) The centroid of T is denoted $(\bar{x}, \bar{y}, \bar{z})$. Symmetry considerations show that $\bar{x} = \bar{y} = 0$. Determine $\bar{z}$.

Part II ("multiple choice" exercises)

Exercise 9 (7%)

A body T covers exactly the area in space that is given in spherical coordinates as

$$\{(\rho, \phi, \theta) : 0 \leq \phi \leq \pi/2, 0 \leq \rho \leq 4\}.$$

The density of T is $\delta(x, y, z) = z$. Which of the 4 iterated integrals below can be used to determine the mass of T.

- ☐ $\int_0^{2\pi} \int_0^{\pi/2} \int_0^4 \rho^2 \cos\phi \sin\phi \, d\rho \, d\phi \, d\theta.$
- ☐ $\int_0^{2\pi} \int_0^{\pi/2} \int_0^4 \rho^3 \cos\phi \sin\phi \, d\rho \, d\phi \, d\theta.$
- ☐ $\int_0^{2\pi} \int_0^{\pi/2} \int_0^4 \rho^3 \sin\phi \, d\rho \, d\phi \, d\theta.$
- ☐ $\int_0^{2\pi} \int_0^{\pi/2} \int_0^4 \rho^2 \cos\phi \, d\rho \, d\phi \, d\theta.$

Exercise 10 (6%)

Consider a complex polynomial $p(z)$ of degree 8 with *real coefficients*. Assume that $p(z)$ can be factorized as

$$p(z) = (z - a)q(z),$$

where $a \in \mathbb{R}$. Mark all correct statements below.

- ☐ $q(z)$ can *always* be factorized as a product consisting only of real 1. and 2. degree polynomials
- ☐ $q(z)$ always contains at least one linear real factor
- ☐ $q(z)$ can be factorized using only real linear factors
- ☐ It is not possible to determine if $q(z)$ has a real root without knowing the coefficients of $p(z)$.

Exercise 11 (6%)

Consider a function $f(x, y)$ of two variables defined on $\mathbf{R}^2$. We are told that all directional derivatives $D_{\mathbf{u}} f(P)$ exist in the point $P(a, b)$. Mark all correct statements below.

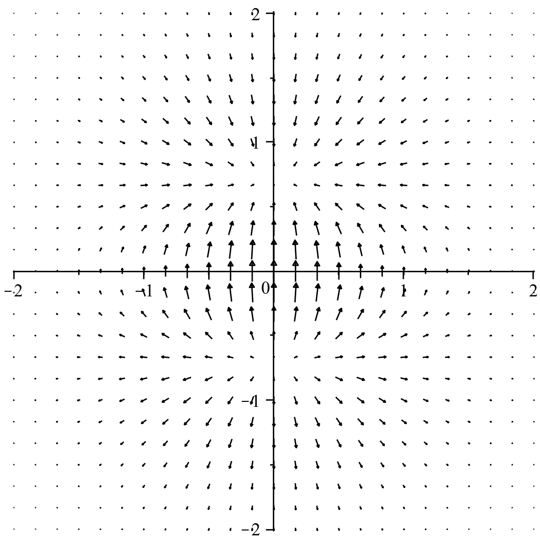
☐ f is continuous in $P(a, b)$
☐ The partial derivatives f_x and f_y exist in a neighborhood of $P(a, b)$
☐ The partial derivatives f_x and f_y exist in $P(a, b)$
☐ From the given information it is not possible to conclude that f is differentiable $P(a, b)$.

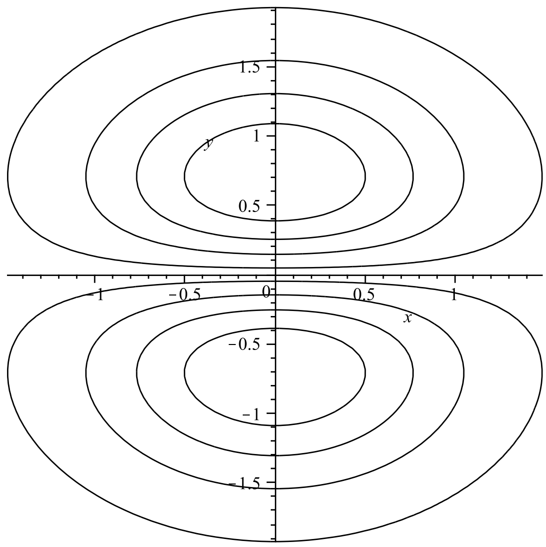
Exercise 12 (6%)

A function $f(x,y)$ is defined on the square

$$R = \{(x,y) : -2 \leq x, y \leq 2\}.$$

The two figures below show certain gradient vectors and level curves, respectively, of the function on R. The function has two critical points in R with coordinates $\pm(0, \pm 1/\sqrt{2})$. Use the figures to determine the nature of each of the critical points and mark the answer below.





(a) The point $(0, 1/\sqrt{2})$ is a

- ☐ local maximum
- ☐ local minimum
- ☐ saddle point.

(b) The point $(0, -1/\sqrt{2})$ is a

- ☐ local maximum
- ☐ local minimum
- ☐ saddle point.