Re-Exam in Calculus

Monday, August 11, 2014

First Year at The TEK-NAT Faculty and Health Faculty

The present exam consists of 7 numbered pages with a total of 12 exercises.

It is allowed to use books, notes, etc. It is **not** allowed to use electronic devices.

The listed percentages specify by which weight the individual exercises influence the total examination.

This exam set has two independent parts.

- Part I contains "regular exercises". Here it is important that you explain the idea behind the solution, and that you provide relevant intermediate results.
- Part II is "multiple choice" exercises. The answers for Part II must be given on these sheets

Remember to write your full name (including middle names), student number together with the course number on each side of your answers. **Number each page and write the total number of pages on the front page of the answers.**

Good luck!

NAME: ______________________________

STUDENT NUMBER: ______________________________

Part I ("regular exercises")

Excercise 1 (8%)

A surface $\mathscr{F}$ is given implicitly as the solution to the equation

$$F(x, y, z) = x^2 + 4y^2 - 8y + z^2 = 0.$$

(a) Verify that the point $P(0, 1, -2)$ belongs to $\mathscr{F}$.

(b) Determine an equation for the tangent plane to $\mathscr{F}$ through $P(0, 1, -2)$.

Excercise 2 (7%)

Calculate the Taylor polynomial of degree 3 for

$$f(x) = 1 + x + \sin(x),$$

about $a = 0$.

Excercise 3 (12%)

(a) Find the general solution to the differential equation

$$y'' - 3y' + 2y = 0.$$

(b) Solve the initial value problem

$$y'' - 3y' + 2y = 0, \qquad y(0) = 2, y'(0) = 1.$$

Excercise 4 (8%)

Find all complex roots of the polynomial

$$p(z) = z^2 - (2+i)z + 2i.$$

Excercise 5 (12%)

A thin plate covers the region

$$\mathscr{R} = \{(x, y) \in \mathbb{R}^2 : 1 \le x^2 + y^2 \le 9,\ y \ge 0\}$$

in the plane. The plate has density $\delta(x, y) = x^2 + y^2$.

Calculate the mass of the plate.

Excercise 6 (12%)

Consider the function

$$f(x, y) = \sin(y) + x + \cos(x) - 2y, \qquad (x, y) \in \mathbb{R}^2.$$

(a) Find the gradient $\nabla f(x, y)$.

(b) The function $g(x)$ is given implicitly by $f(x, g(x)) = 1$. Determine $g(0)$ and $g'(0)$.

Excercise 7 (5%)

Consider the function

$$f(x,y) = \arccos(x^2 + y^2).$$

(a) Find the domain of f.

(b) Calculate the partial derivatives $f_x(x,y)$ og $f_y(x,y)$.

Excercise 8 (8%)

A curve in space is given by

$$\mathbf{r}(t) = \cos(2t)\,\mathbf{i} + \sin(2t)\,\mathbf{j} + \sqrt{5}t\mathbf{k}, \qquad t \in [-1, 2\pi].$$

Calculate the arc length of the curve from $t = 0$ to $t = \pi$.

Part II ("multiple choice" exercises)

Exercise 9 (10%).

Answer the following 6 true/false exercises:

a. We have the identity

$$\sin(\alpha+\beta) = 2\cos(\alpha)\sin(\beta)$$

for *all* $\alpha \in \mathbb{R}$.

☐ True ☐ False

b. A continuously differentiable function $f : \mathbb{R}^2 \to \mathbb{R}$ satisfying $\nabla f(P) = \mathbf{0}$ for some $P \in \mathbb{R}^2$ always has a local extrema in P.

☐ True ☐ False

c. For a complex number $z \neq 0$ it holds true that

$$\frac{1}{z^2} = \frac{\bar{z}^2}{|z|^4}.$$

☐ True ☐ False

d. The following identity is satisfied by the inverse tangent function:

$$\arctan(\tan(x)) = x, \qquad x \in (-\pi/2, \pi/2).$$

☐ True ☐ False

e. A real-value function f defined on a region $\mathscr{R}$ in the xy-plane has a global maximum at $(a,b) \in \mathscr{R}$ if $f(a,b) \geq f(x,y)$ for all $(x,y) \in \mathscr{R}$.

☐ True ☐ False

f. We have the identity

$$\frac{d}{dx}\arctan(\sqrt{x}) = \frac{1}{1+x}$$

for *all* $x > 0$.

☐ True ☐ False

Notice. In exercises 10–12 the evaluation is performed using the following principle: Every false mark cancels one true mark. Therefore, one gains *nothing* from a "safe bet".

Excercise 10 (6%)

A body T covers the simple region in space given by

$$\{(x, y, z) : \quad (x, y) \in R, 0 \le z \le x^2 + y^2\},$$

with

$$R = \{(x, y) : \quad 0 \le x \le 1,\, x^2 \le y \le \sqrt{x}\}.$$

The density of T is $\delta(x, y, z) = 1+z$. Mark all correct statements below (note: The value of the integral should *not* be computed).

☐ The mass of T can be computed as follows: $m = \displaystyle\int_0^1 \int_{x^2}^{\sqrt{x}} \int_0^{x^2+y^2} (1+z)\, dz dy dx$

☐ The volume of T can be computed as follows: $V = \displaystyle\int_0^1 \int_{x^2}^{\sqrt{x}} \int_0^{x^2+y^2} (1+z)\, dz dy dx$

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Excercise 11 (5%)

Let $f(x, y)$ be a real-valued continuous function defined on a region R in the xy-plane. R consists of the points on and inside a simple closed curve C. Mark all correct statements below.

☐ f has a global extrema on R.

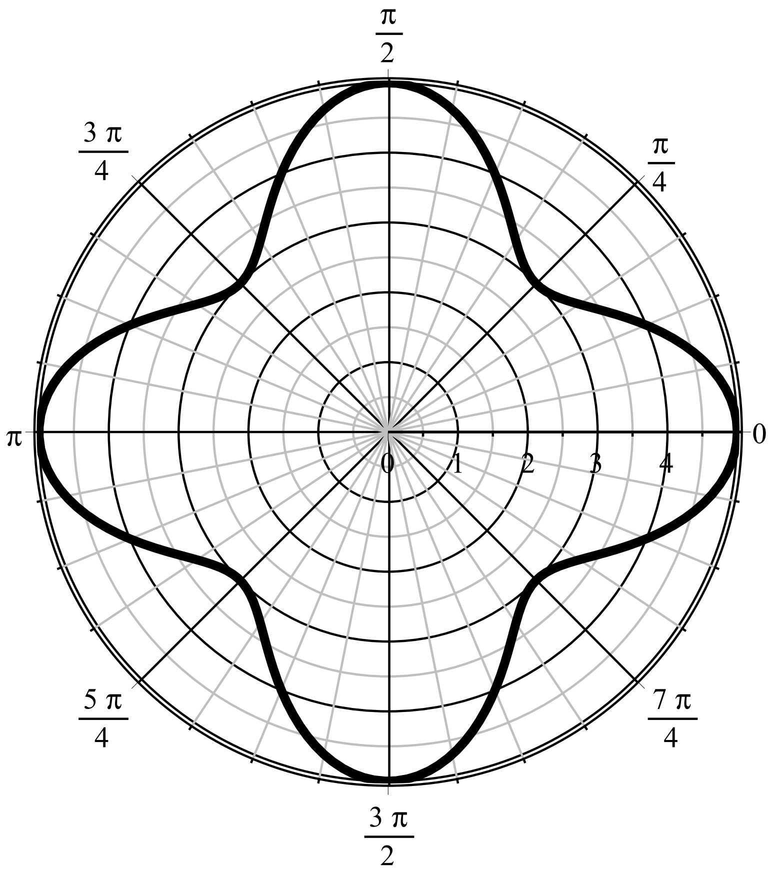
☐ f has a global extrema on the boundary C.

☐ If f has a tangent plane at $\mathbf{a} \in R$ parallel to the xy-plane, then $\mathbf{a}$ is called a critical point of f.

☐ If f has a local extrema at $\mathbf{a} \in R$, then the partial derivatives of f exist at $\mathbf{a}$.

Exercise 12 (7%)

The figure below shows the graph of the function $r = f(\theta)$ displayed in polar coordinates



Which of the expressions given below for f, along with the domain for the θ, correspond to the above figure.

- ☐ $f(\theta) = 2 + \sin(4\theta),\ 0 \le \theta \le 2\pi$
- ☐ $f(\theta) = 4 + \cos(4\theta),\ 0 \le \theta \le 2\pi$
- ☐ $f(\theta) = 2 + \cos(4\theta),\ 0 \le \theta \le \pi$
- ☐ $f(\theta) = 3 + 2\sin(2\theta),\ 0 \le \theta \le 2\pi$
- ☐ $f(\theta) = 2 + \sin(2\theta),\ 0 \le \theta \le 2\pi$
- ☐ $f(\theta) = 5\cos(2\theta),\ 0 \le \theta \le \pi.$