Re-exam in Calculus

Thursday August 16 2012

First Year at The TEK-NAT Faculty and Health Faculty

The present exam consists of 7 numbered pages with a total of 12 exercises.

It is allowed to use books, notes, etc. It is **not** allowed to use electronic devices.

The listed percentages specify by which weight the individual exercises influence the total examination.

This exam set has two independent parts.

- Part I contains "regular exercises". Here it is important that you explain the idea behind the solution, and that you provide relevant intermediate results.
- Part II is "multiple choice" exercises. The answers for Part II must be given on these sheets

Remember to write your full name (including middle names), student number together with the course number on each side of your answers. **Number each page and write the total number of pages on the front page of the answers.**

Good luck!

NAME: ____________________

STUDENT NUMBER: ____________________

COURSE NUMBER: ____________________

Part I ("regular exercises")

Exercise 1 (8%)

A surface $\mathscr{F}$ is given implicitly as the solution of the equation

$$F(x, y, z) = 2x + 2\ln(2y) + z^2 - 9 = 0.$$

(a) Determine the gradient vector $\nabla F(x, y, z)$.

(b) Determine an equation for the tangent plane to $\mathscr{F}$ through the point $(4, \frac{1}{2}, 1)$.

Exercise 2 (16%)

(a) Find the uniquely determined solution to the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0,$$

that fulfills

$$y(0) = 0 \quad \text{and} \quad y'(0) = 1.$$

(b) Find a particular solution to the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = x.$$

(c) Find the complete solution to the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = x.$$

Exercise 3 (8%)

Solve the complex 2. degree equation

$$z^2 - (1+i)z + 2 + 2i = 0.$$

Exercise 4 (8%)

The area $\mathscr{R}$ in the xy plane is given by

$$\mathscr{R} = \{(x,y) : 0 \leq y \leq 1, -y \leq x \leq y\}.$$

Find the double integral of $f(x,y) = x^2$ over the area $\mathscr{R}$.

Exercise 5 (8%)

Compute the Taylor polynomial of degree 3 for

$$f(x) = \cos(2x),$$

around $a = 0$.

Exercise 6 (10%)

A thin plate covers exactly the area

$$\mathscr{R} = \{(x, y) \in \mathbb{R}^2 : 1 \le x^2 + y^2 \le 4,\ x \ge 0,\ y \ge 0\}$$

in the xy plane. The density of the plate is $\delta(x, y) = y$.

Determine the mass of the plate.

Exercise 7 (5%)

Consider the function

$$f(x, y) = \sin(x + y^4).$$

(a) Determine the partial derivatives $f_x(x, y)$ and $f_y(x, y)$.

(b) Determine the differential df.

Exercise 8 (5%)

Consider the planar curve

$$\begin{aligned} x &= t \\ y &= 2t^2, \end{aligned}$$

where $t \in \mathbb{R}$. Compute the curvature $\kappa(t)$ of the curve for $t \in \mathbb{R}$.

Part II (”multiple choice” exercises)

Notice. In exercises 9 and 10 the evaluation is performed using the following principle: Every false mark cancels one true mark. Therefore, one gains *nothing* from a “safe bet”.

Exercise 9 (6%)

A body T covers the area in space that is given by

$$\{(x,y,z): \quad -1 \le y \le 2, \quad 0 \le x \le 4, \quad -x^2 \le z \le 1+y^2\}.$$

The density of T is $\delta(x,y,z) = 1+y^2$. Which of the following 4 iterated integrals can be used to evaluate the mass of T (note: The value of the integral should *not* be computed)

☐ $\int_{-1}^{2}\int_{0}^{4}\int_{-x^2}^{1+y^2} (1+y^2)\, dy\, dx\, dz.$

☐ $\int_{-1}^{2}\int_{0}^{4}\int_{1+y^2}^{-x^2} (1+y^2)\, dz\, dy\, dx.$

☐ $\int_{-1}^{2}\int_{0}^{4}\int_{-x^2}^{1+y^2} (1+y^2)\, dz\, dx\, dy.$

☐ $\int_{0}^{4}\int_{-1}^{2}\int_{-x^2}^{1+y^2} (1+y^2)\, dz\, dx\, dy.$

Exercise 10 (5%)

Consider two complex numbers $c_1 = r_1 e^{i\theta_1}$ and $c_2 = r_2 e^{i\theta_2}$, where $r_1, r_2, \theta_1, \theta_2 \in \mathbb{R}$. Mark all correct statements below

☐ $c_1 c_2 = r_1 r_2 e^{i(\theta_1+\theta_2)}$.

☐ $\overline{c_1} c_2 = r_1 r_2 e^{i(\theta_2-\theta_1)}$

☐ $\frac{c_1}{c_2} = \frac{r_1}{r_2} e^{i(\theta_1+\theta_2)}$, provided that $r_2 \neq 0$.

☐ $c_1^7 = r_1^7 e^{i\theta_1}$.

Exercise 11 (14%).

Answer the following 5 true/false exercises

a. Consider a continuous function f defined on an area $\mathscr{R}$ in the plane, where $\mathscr{R}$ is given by

$$\mathscr{R} = \{(x, y) : c \le y \le d, \quad x_1(y) \le x \le x_2(y)\},$$

where x_1 and x_2 are continuous functions and $c \le d$ are constants. The corresponding double integral can be evaluated as the following iterated integral.

$$\iint_{\mathscr{R}} f(x, y)\, dA = \int_c^d \int_{x_1(y)}^{x_2(y)} f(x, y)\, dx\, dy.$$

☐ True ☐ False

b. A real function f defined on an area $\mathscr{R}$ in the xy plane, has a global minimum in $(a, b) \in \mathscr{R}$ if $f(a, b) \ge f(x, y)$ for all $(x, y) \in \mathscr{R}$.

☐ True ☐ False

c. Consider a differentiable function $f(x, y, z)$ defined on $\mathbb{R}^3$. If $\nabla f(0,0,0) = \mathbf{0}$, then f has a local extrema in the point $(0,0,0)$.

☐ True ☐ False

d. The domain for $f(x) = \arccos(x)$ is $[0, 1]$.

☐ True ☐ False

e. For every $y_0 \in \mathbb{R}$ the equation in x, x,

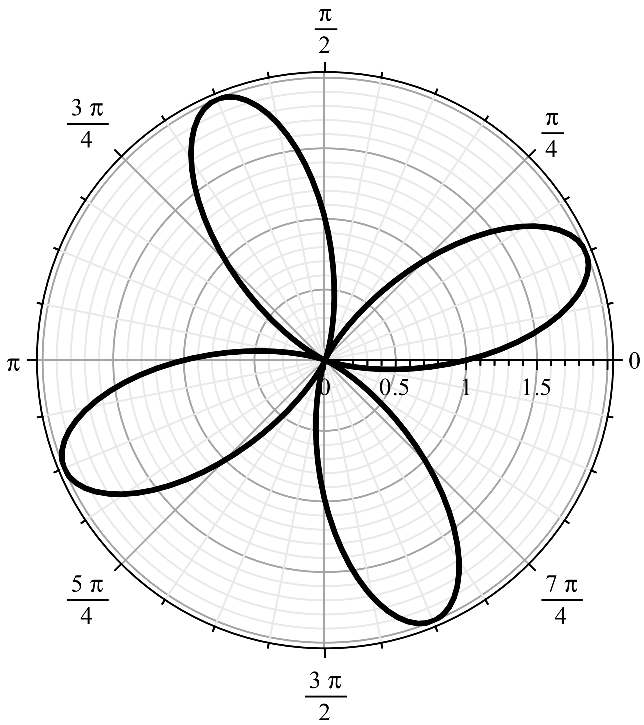
$$\arctan(x) = y_0,$$

has at least one solution.

☐ True ☐ False

Exercise 12 (7%)

The figure below shows the graph of the function $r = f(\theta)$ displayed in polar coordinates



Which of the below expressions for f along with the domain for the θ gives the above figure.

- ☐ $f(\theta) = 1 + \cos(4\theta),\ 0 \le \theta \le 2\pi$
- ☐ $f(\theta) = 2 - \cos(2\theta),\ 0 \le \theta \le \pi$
- ☐ $f(\theta) = 1 + \sin(4\theta),\ 0 \le \theta \le \pi$
- ☐ $f(\theta) = 1 + \sin(2\theta),\ 0 \le \theta \le 2\pi$
- ☐ $f(\theta) = 1 + 2\cos(2\theta),\ 0 \le \theta \le \pi$
- ☐ $f(\theta) = 1 + \sin(4\theta),\ 0 \le \theta \le 2\pi$.