For at finde den danske version af prøven, begynd i den modsatte ende!

Please disregard the Danish version on the back if you participate in this English version of the exam.

Exam in Calculus

First Year at the Technical Faculty for IT and Design and the Faculty of Engineering and Science

August 22, 2017, 9:00 – 13:00

This test consists of 8 pages and 12 problems. All problems are “multiple choice” problems. Your answers must be given on these sheets.

It is allowed to use books, notes, xerox copies etc. It is **not allowed** to use any electronic devices such as pocket calculators, mobile phones or computers.

The listed percentages specify by which weight the individual problems influence the total examination.
Problems 6(a), 9 and 11(c) may have more than one correct answer.
Your marks concerning these problems will be evaluated as follows: every wrong mark will annul one correct mark.

Remember to write your full name (including middle names) together with your student number below.
Moreover, please mark the team that you participate in.

Good luck!

NAME: ____________________

STUDENT NUMBER: ____________________

- ☐ Team BBT (København) — Bedia Møller
- ☐ Team 1: LAND – ST — Horia Cornean
- ☐ Team 3: MAT – MAOK – MATTEK — Nikolaj Hess-Nielsen
- ☐ Team L (København) — Iver Ottosen

Problem 1 (8 points)

A function f is given by

$$f(x) = e^{(x^2)}$$

for a real parameter x.

(a) (4 points) Which of the following functions agrees with the second derivative $f''(x)$?

☐ $2e^{(x^2)}$ ☐ $(2+4x^2)e^{(x^2)}$ ☐ $2xe^{2x}$

☐ $4x^2e^{(x^2)}$ ☐ e^2 ☐ $e^{(x^2)}$

(b) (4 points) One of the following polynomials coincides with the second order Taylor polynomial for the function f at $x = 1$. Which?

☐ $2e(x-1)+3e(x-1)^2$ ☐ $e+2e(x-1)+3e(x-1)^2$

☐ $e+2ex+3ex^2$ ☐ $e+2e(x-1)+6e(x-1)^2$

Problem 2 (8 points)

A planar curve is given by

$$x = t^2,$$
$$y = 2t^3;$$

the parameter t can take any real value.

(a) (2 points) For which value of the parameter t does the curve pass through the point $P = (1, 2)$?

☐ -1 ☐ 0 ☐ 1 ☐ 2

(b) (2 points) Which of the following vectors is the velocity vector at P?

☐ $\begin{bmatrix} 2 \\ 12 \end{bmatrix}$ ☐ $\begin{bmatrix} 2 \\ 6 \end{bmatrix}$ ☐ $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ ☐ $\begin{bmatrix} -12 \\ 2 \end{bmatrix}$ ☐ $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(c) (4 points) Which of the following numbers agrees with the radius of curvature $\rho(P) = \frac{1}{\kappa(P)}$ at P?

☐ $\frac{3\sqrt{10}}{200}$ ☐ $\frac{2\sqrt{10}}{3}$ ☐ 0

☐ $\frac{20\sqrt{10}}{3}$ ☐ $2\sqrt{37}$ ☐ intet af dem

Problem 3 (6 points)

A space curve is given by

$$x = t,$$
$$y = \frac{\sqrt{6}}{2} t^2,$$
$$z = t^3,$$

the parameter t can take any positive real value.

(a) (3 points) Mark the correct expression for the speed $v(t)$.

- ☐ $1 + 3t^2$
- ☐ $1 + \sqrt{6}t + 3t^2$
- ☐ $\sqrt{1 + 3t^2 + 9t^4}$
- ☐ $1 + 6t^2 + 9t^4$

(b) (3 points) Which of the following agrees with the arc length between $t = 1$ and $t = 2$?

☐ $4\frac{4}{5}$ ☐ 2 ☐ -1 ☐ 8 ☐ 10

Problem 4 (8 points)

The function f is defined in the first quadrant ($x > 0, y > 0$) by

$$f(x, y) = x + 8y + \frac{1}{xy}.$$

The graph of this function opens upward, i.e., its values tend to ∞ when x or y approach 0 or grow beyond all limits.

(a) (4 points) One of the following is a critical point for the function f. Which?

- ☐ $(0, 0)$
- ☐ $(\frac{1}{2}, 4)$
- ☐ $(1, 1)$
- ☐ $(2, \frac{1}{4})$
- ☐ $(-2, -\frac{1}{4})$
- ☐ $(\frac{1}{4}, 2)$

(b) (4 points) Mark the true ones among the following assertions:

☐ The critical point is a saddle point.

☐ The function takes the value 6 at the critical point. This value is the global minimum for the function f.

☐ The function takes the value 33 at the critical point. This value is the global minimum for the function f.

☐ The function takes the value 6 at the critical point. This value is the global maximum for the function f.

☐ The function takes the value 6 at the critical point. This value is a local but not a global minimum.

Problem 5 (7 points)

A function is given by

$$f(x,y) = \frac{y}{x^2}.$$

(a) (2 points) Mark whether the domain of the function f consists of all points (x,y) in the plane *apart from*

- ☐ the origin
- ☐ the diagonal $y = x$
- ☐ the x-axis
- ☐ the y-axis
- ☐ the parabola $y = x^2$

(b) (3 points) Which of the following fits with the level curve $f(x,y) = 2$?

☐ A straight line with slope 2 through the origin.

☐ A parabola through the origin that opens towards positives.

☐ A parabola through the origin that opens towards positives – apart from its vertex at the origin.

☐ A parabola through the origin that opens towards negatives – apart from its vertex at the origin.

☐ A circle with radius 1 centered at the origin.

(c) (2 points) Which of the following fits with the level curve $f(x,y) = 0$?

- ☐ an ellipse
- ☐ the x-axis
- ☐ the x-axis apart from the origin
- ☐ the y-axis
- ☐ the y-axis apart from the origin
- ☐ the empty set

Problem 6 (8 points)

A region L in 3D-space is bounded by the triangle in the XY-plane given by $x \leq 0, y \geq 0$ and $y \leq x + 1$, and furthermore the surface given by the equation $z = 1 - x^2 - y^2$.

(a) (4 points) Which of the following integral expressions agree with the volume of the region L?

- ☐ $\int_{-1}^{0}(1 - x^2 - y^2)\,dy\,dx$
- ☐ $\int_{-1}^{0}\int_{0}^{1+x}(1 - x^2 - y^2)\,dy\,dx$
- ☐ $\int_{0}^{1}\int_{y-1}^{0}(1 - x^2 - y^2)\,dx\,dy$
- ☐ $\int_{0}^{1}\int_{0}^{1+x}(1 - x^2 - y^2)\,dy\,dx$

(b) (4 points) What is the volume of the region L?

☐ 3 ☐ 1 ☐ $\frac{1}{3}$ ☐ $\frac{1}{6}$ ☐ $\frac{1}{24}$ ☐ $-\frac{1}{3}$

Problem 7 (8 points)

A body T in 3D-space in the shape of a tetrahedron T is bounded by four planes; it can be described by $x \geq 0, y \geq 0, z \geq 0$ and $x + y + z \leq 1$, and it has a density given by $\delta(x, y, z) = x$. Which of the following numbers coincides with the mass of the body T?

☐ 1 ☐ $\frac{1}{8}$ ☐ $-\frac{1}{24}$

☐ $\frac{1}{3}$ ☐ $\frac{1}{24}$ ☐ $\frac{1}{48}$

Problem 8 (14 points)

A function is given by

$$f(x, y) = \ln(y - x^2).$$

(a) (2 points) Mark whether the function's domain consists of all points satisfying

☐ $y < x^2$ ☐ $y > x^2$

☐ $y \geq x^2$ ☐ $(x, y) \neq (0, 0)$

(b) (3 points) Which of the following expressions corresponds to the second order partial derivative $f_{xy}(x, y)$?

☐ $-2x \ln(y - x^2)$ ☐ $-2x$

☐ $\frac{-1}{(y-x^2)^2}$ ☐ $\frac{2x}{(y-x^2)^2}$

(c) (3 points) Which of the following vectors coincides with the function's gradient vector ∇f at the point $P = (1, 2)$?

☐ $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ ☐ $\begin{bmatrix} \frac{-2}{3} \\ \frac{1}{3} \end{bmatrix}$

☐ $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ ☐ $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(d) (3 points) Which of the following numbers agrees with the directional derivative $D_{\mathbf{u}} f(P)$ at the point $P = (1, 2)$ in the direction determined by the unit vector $\mathbf{u} = 0.8\mathbf{i} - 0.6\mathbf{j} = (0.8, -0.6)$?

☐ 1 ☐ 0 ☐ -2.2 ☐ 2 ☐ -1

(e) (3 points) Which of the following equations determines the surface's tangent plane at $Q = (1, 2, 0)$?

☐ $z = -2x + y$ ☐ $z = -2(x - 2) + y - 1$

☐ $z = -2x + y + \ln(4)$ ☐ none of these

Problem 9 (10 points)

A surface $\mathcal{F}$ in 3D space is given implicitly by the equation

$$F(x, y, z) = x^2 + y^2 - z^2 = 1.$$

(a) (2 points) Which of the following points are contained in the surface $\mathcal{F}$?

- ☐ $(1, 0)$
- ☐ $(1, 0, 0)$
- ☐ $(1, 1, 1)$
- ☐ $(2, 2, -3)$
- ☐ $(3, 4, 5)$
- ☐ $(-1, -1, 0)$

(b) (2 points) Which of the following vectors are perpendicular to $\mathcal{F}$'s tangent plane at $P = (1, 1, -1)$?

- ☐ $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$
- ☐ $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$
- ☐ $\begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$
- ☐ $\begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix}$

(c) (2 points) Which of the following points are contained in $\mathcal{F}$'s tangent plane at $P = (1, 1, -1)$?

- ☐ $(1, 1, 0)$
- ☐ $(1, 0, 0)$
- ☐ $(2, 2, 3)$
- ☐ $(2, 2, -3)$

(d) (4 points) At some of the following points Q on $\mathcal{F}$ the tangent plane to $\mathcal{F}$ at the point Q is parallel to the plane $z = x + y$? Which of them?

- ☐ $Q = (1, 1, 1)$
- ☐ $Q = (1, 1, -1)$
- ☐ $Q = (1, 0, 0)$
- ☐ $Q = (0, 0, 1)$
- ☐ $Q = (-1, -1, -1)$
- ☐ $Q = (-1, 1, 1)$

Problem 10 (8 points)

A complex number z has polar form $\sqrt{2}e^{\frac{\pi i}{4}}$.

(a) (2 points) Which of the following agrees with z in standard form?

☐ $\sqrt{2}+\sqrt{2}i$ ☐ i ☐ $1+i$ ☐ none of these

(b) (2 points) Which of the following agrees with $z\bar{z}$ in polar form?

☐ 2 ☐ $2e^{\pi i}$ ☐ $\sqrt{2}e^{\frac{\pi i}{2}}$ ☐ ingen af dem

(c) (2 points) Which of the following agrees with $\frac{\bar{z}}{z}$ in standard form?

☐ i ☐ $\sqrt{2}-\sqrt{2}i$ ☐ $-i$ ☐ none of these

(d) (2 points) Which of the following agrees with $\frac{\bar{z}}{z}$ in polar form?

☐ $e^{\frac{\pi i}{2}}$ ☐ $2e^{\frac{3\pi i}{4}}$ ☐ $e^{\frac{3\pi i}{2}}$ ☐ none of these

Problem 11 (9 points)

A second order homogeneous differential equation is given by

$$y'' - 6y' + 9y = 0.$$

(a) (3 points) The list below contains a number of function expressions including arbitrary constants c_1 and c_2. Mark the expression that describes all solutions of the differential equation.

☐ $y(t) = c_1e^{3t}cos(t) + c_2e^{3t}\sin(t)$ ☐ $y(t) = c_1e^{3t}\cos(t^2) - c_2e^{3t}\sin(t^2)$
☐ $y(t) = c_1e^{t} + c_2e^{9t}$ ☐ $y(t) = c_1t^3 + c_2t^{-3}$
☐ $y(t) = c_1e^{3t} + c_2te^{3t}$ ☐ $y(t) = c_1e^{3t}cos(2t) + c_2e^{3t}\sin(2t)$

(b) (3 points) The differential equation has a unique solution $y(t)$ with initial conditions $y(1) = 3e^3,\ y'(1) = 10e^3$. Mark the value $y(0)$ of that solution at $t = 0$.

☐ 6 ☐ 2 ☐ 0 ☐ none of these

(c) (3 points) Which of the following function expressions is a (particular) solution of the inhomogeneous differential equation

$$y'' - 6y' + 9y = 9t + 3?$$

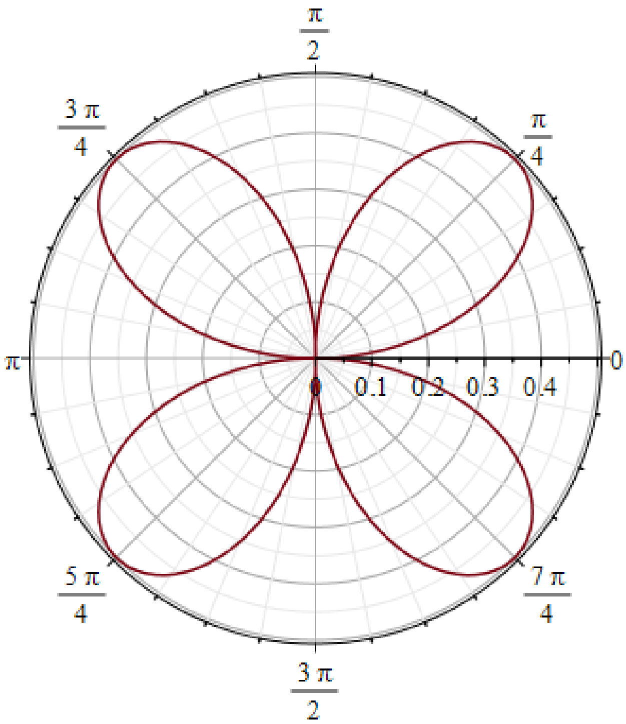
☐ $t+1$ ☐ $e^{3t}+t+1$
☐ $t^2+\frac{t}{2}-1$ ☐ $-2te^{3t}+t+1$
☐ $9t+\frac{17}{3}$ ☐ $e^{3t}-te^{3t}$

Problem 12 (6 points)

The figure below shows the graph of a function

$$r = f(\theta), \quad 0 \le \theta \le 2\pi.$$

in polar coordinates.



Which of the functions below gives rise to that graph?

- ☐ $f(\theta) = \sin(\theta)$
- ☐ $f(\theta) = (\sin(\theta))^2$
- ☐ $f(\theta) = (\cos(\theta))^2$
- ☐ $f(\theta) = \sin(2\theta)$
- ☐ $f(\theta) = \frac{\sin(2\theta)}{2}$
- ☐ $f(\theta) = \frac{\sin(\theta)}{2}$