Reexam in Calculus

First Year at the Faculty of Engineering and Science
and the Technical Faculty of IT and Design

21 February 2018

The present exam set consists of 8 numbered pages with 12 multiple-choice problems. A number of points are specified for each question. The total number of points equals 100.

It is allowed to use books, notes etc. It **is not** allowed to use electronic devices.

Your answers must be marked directly in this exam set. The grading of the exam is based solely on these marks. Remember to write your **full name** and **student number** below.

Good luck!

NAME: ______________________________

STUDENT NUMBER: ______________________________

Answers

Problem 1 (13 points)

(a) (5 points). A second order differential equation is given by

$$y'' - 10y' + 25y = 0.$$

Below is a list of function expressions containing two arbitrary constants c_1 and c_2. Mark the expression which constitute the general solution of the differential equation.

- ☐ $y(t) = c_1 e^{-3t} + c_2 e^{t}$
- ☐ $y(t) = c_1 e^{-5t} + c_2 e^{2t}$
- ☐ $y(t) = c_1 e^{t} + c_2 e^{5t}$
- ☑ $y(t) = c_1 e^{5t} + c_2 t e^{5t}$
- ☐ $y(t) = c_1 e^{2t} + c_2 t e^{2t}$
- ☐ $y(t) = c_1 e^{2t} \cos(5t) + c_2 e^{2t} \sin(5t)$
- ☐ $y(t) = c_1 e^{t} \cos(6t) + c_2 e^{t} \sin(6t)$
- ☐ $y(t) = c_1 e^{2t} \cos(3t) + c_2 e^{2t} \sin(3t)$

(b) (4 points). Mark the solution to the initial value problem

$$y'' - 10y' + 25y = 0, \quad y(0) = 2, \quad y'(0) = 7$$

among the following options:

- ☐ $y(t) = -\frac{5}{4} e^{-3t} + \frac{13}{4} c_2 e^{t}$
- ☐ $y(t) = 3e^{-5t} - c_2 e^{2t}$
- ☐ $y(t) = -\frac{1}{4} e^{t} + \frac{5}{4} e^{5t}$
- ☑ $y(t) = 2e^{5t} - 3te^{5t}$
- ☐ $y(t) = 2e^{2t} + 3te^{2t}$
- ☐ $y(t) = 2e^{2t} \cos(5t) + 3e^{2t} \sin(5t)$
- ☐ $y(t) = 2e^{t} \cos(6t) + 4e^{t} \sin(6t)$
- ☐ $y(t) = e^{2t} \cos(3t) + e^{2t} \sin(3t)$

(c) (4 points). Consider the inhomogeneous differential equation

$$y'' - 10y' + 25y = 5t + 3.$$

Which one of the following functions is a particular solution to that equation?

- ☐ $\frac{5}{2}t + \frac{3}{2}$
- ☑ $\frac{1}{5}t + \frac{1}{5}$
- ☐ $t - 1$
- ☐ $5t - 1$
- ☐ $t^2 - 5t - 2$
- ☐ $2t^2 - t + 1$
- ☐ $2e^{t}$
- ☐ $3t - 4$

Problem 2 (7 points)

A plane curve is given by

$$x = t - e^t,$$
$$y = t + e^t,$$

where the parameter t runs through the real numbers.

(a) (3 points). The point $P = (-1, 1)$ lies on the curve. Which value of the parameter t corresponds to this point?

☐ -2	☐ -1	☐ 1	☐ 3
☐ $-\ln(2)$	☑ 0	☐ $\ln(2)$	☐ $\frac{7}{2}$

(b) (4 points). What is the curvature of the curve for $t = 0$?

☐ $\frac{5}{4}$	☐ $\frac{\sqrt{3}}{4}$	☐ $\frac{1}{5}$	☐ 8
☐ $\frac{\sqrt{2}}{4}$	☐ $\sqrt{5}$	☐ $\frac{5}{3}$	☑ $\frac{1}{4}$

Problem 3 (8 points)

A curve in space is given by

$$x = \tfrac{1}{2}t,$$
$$y = \tfrac{2}{3}t^{\frac{3}{2}},$$
$$z = \tfrac{1}{2}t^2,$$

where the parameter t runs through the positive real numbers.

(a) (3 points). What is the derivative y'?

☐ $t^3 + 1$	☐ $t\ln(t)$	☐ t	☐ $t\sqrt{t}$
☐ $\frac{1}{3t}$	☐ t^2	☑ $\sqrt{t}$	☐ $\frac{4}{15}t^{\frac{5}{2}}$

(b) (5 points). What is the arc length of the curve from $t = 1$ to $t = 5$?

☑ 14	☐ $10\sqrt{2}$	☐ $7\sqrt{3}$	☐ 18
☐ 5	☐ 8	☐ 11	☐ $\frac{25}{2}$

Problem 4 (7 points)

A function is defined by

$$f(x) = \frac{1}{x^2 + 2}.$$

(a) (3 points). What is the derivative $f'(x)$?

- ☐ $\frac{1}{2x}$
- ☐ $(x^2+2)^{-\frac{1}{2}}$
- ☐ $2x(x^2+2)^{\frac{1}{2}}$
- ☑ $-\frac{2x}{(x^2+2)^2}$
- ☐ $-\frac{1}{(x^2+2)^2}$
- ☐ $\ln(x^2+2)$

(b) (4 points). Which one of the polynomials below is the second order Taylor polynomial for f at the points $x = 0$?

- ☐ $1 + x - \frac{3}{2}x^2$
- ☐ $\frac{1}{2} + x - \frac{1}{2}x^2$
- ☐ $2 + 4x - 5x^2$
- ☐ $\frac{1}{2} + \frac{1}{2}x + \frac{1}{6}x^2$
- ☑ $\frac{1}{2} - \frac{1}{4}x^2$
- ☐ $x - \frac{1}{2}x^2$

Problem 5 (7 points)

Consider the differential equation

$$\frac{dy}{dx} = \frac{e^x}{y}, \quad y > 0.$$

There is a unique solution $y(x)$ with initial value $y(0) = 2$. Answer the following questions regarding this solution:

(a) (3 points). What is the derivative $y'(0)$?

- ☐ -2
- ☑ $\frac{1}{2}$
- ☐ 2
- ☐ $-\frac{1}{2}$
- ☐ 1
- ☐ 3

(b) (4 points). What is the function value $y(1)$?

- ☐ 5
- ☐ $\frac{1}{3}$
- ☐ $\ln(2)$
- ☐ $e+1$
- ☐ 1
- ☑ $\sqrt{2e+2}$

Problem 6 (6 points)

Two complex numbers are given by

$$z_1 = \frac{4+3i}{2-i} + 1 + i \quad , \quad z_2 = (e^{2+\frac{\pi}{6}i})^3$$

(a) (3 points). What is z_1 written in standard form?

- [x] $2+3i$
- [ ] $\frac{1}{4}+\frac{5}{4}i$
- [ ] $1-i$
- [ ] $8i$
- [ ] $1+i$
- [ ] $\frac{2}{3}+\frac{2}{3}i$
- [ ] $-3i$
- [ ] $-5+i$

(b) (3 points). What is z_2 written in standard form?

- [ ] $1+e^3 i$
- [ ] $1-e^2 i$
- [ ] $-e^9$
- [ ] $e+e^2 i$
- [ ] e^6
- [ ] $1+e^2 i$
- [x] $e^6 i$
- [ ] $1+3i$

Problem 7 (6 points)

A function is defined by

$$f(x,y) = x\cos(y) - y\sin(x).$$

(a) (2 points).What is the partial derivative $f_x(x,y)$?

- [ ] $-\sin(y) - \cos(x)$
- [ ] $\tan(y) + \sin(x)$
- [ ] $\cos(y)$
- [x] $\cos(y) - y\cos(x)$
- [ ] $\tan(y) - 1$
- [ ] $x^2\cos(y)$

(b) (4 points). The graph of f has a tangent plane at the point $P = (\frac{\pi}{2}, 0, f(\frac{\pi}{2}, 0))$. Mark an equation for this plane.

- [x] $x - y - z = 0$
- [ ] $x + 2y - 3z = \frac{\pi}{2}$
- [ ] $2x + y - 2z = 0$
- [ ] $x - y + z = \pi$
- [ ] $x + y - z = \frac{\pi}{2}$
- [ ] $x + y + 2z = -\pi$

Problem 8 (16 points)

A function of two real variables is given by the transformation rule

$$f(x,y) = \frac{y}{1+x^2+y^2}.$$

Answer the questions below regarding this function.

(a) (4 points). The domain of f consists of all those points (x,y) which satisfy

- ☐ $x \geq 0$ and $y \geq 0$
- ☐ $y \neq 0$
- ☐ $x > 0$ and $y > 0$
- ☑ x and y are real numbers
- ☐ $x \neq 0$ and $y \neq 0$
- ☐ $x \neq y$

(b) (4 points). Which of the following points are critical points for f? (*Remark: One false mark will cancel one true mark in this subquestion.*)

- ☑ $(0,1)$
- ☐ $(0,2)$
- ☐ $(1,1)$
- ☑ $(0,-1)$
- ☐ $(1,-1)$
- ☐ $(2,-1)$

(c) (4 points). What is the directional derivative $D_{\mathbf{u}}f(P)$ at the point $P=(1,1)$ and in the direction of the unit vector $\mathbf{u} = \frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j} = \langle \frac{3}{5}, \frac{4}{5} \rangle$?

- ☐ $\frac{8}{5}$
- ☐ 1
- ☐ $\frac{3}{25}$
- ☐ $-\frac{3}{5}$
- ☐ 2
- ☑ $-\frac{2}{45}$

(d) (4 points). The level curve with equation $f(x,y) = \frac{1}{4}$ can be described as:

- ☐ A parabola with equation $y = x^2 + 3$.
- ☐ A parabola with equation $y = x^2 - x + 4$.
- ☐ A straight line through $(0, \sqrt{3})$ with slope 2.
- ☐ A straight line through $(0,3)$ with slope $-\frac{1}{2}$.
- ☑ A circle with center at $(0,2)$ and radius $\sqrt{3}$.
- ☐ A circle with center at $(1,1)$ and radius $\frac{1}{3}$.

Problem 9 (7 points)

A lamina covers a region $\mathcal{R}$ in the plane precisely. The region $\mathcal{R}$ consists of those points (x, y) which satisfy the inequalities

$$0 \leq x, \quad 0 \leq y, \quad 3x + y \leq 6.$$

The density of the lamina is $\delta(x, y) = x$. What is the mass of the lamina?

☐ $\frac{1}{2}$	☐ 2	☐ $\frac{9}{2}$	☐ $5\sqrt{2}$	☐ 12
☐ 1	☑ 4	☐ 5	☐ 10	☐ 14

Problem 10 (10 points)

A region $\mathcal{R}$ in the plane consists of those points (x, y) which satisfy the inequalities

$$0 \leq x \leq y, \quad 1 \leq x^2 + y^2 \leq 9.$$

Mark the value of the double integral

$$\iint_{\mathcal{R}} \frac{y}{x^2 + y^2}\, dA.$$

☐ $\frac{1}{3}$	☑ $\sqrt{2}$	☐ $\frac{7}{2}$	☐ 6	☐ 12
☐ $\frac{1}{2}$	☐ 3	☐ $5\sqrt{2}$	☐ 10	☐ $\frac{25}{2}$

Problem 11 (8 points)

A region $\mathcal{T}$ in space consists of those points (x, y, z) which satisfy the inequalities

$$0 \leq x \leq 2, \quad 0 \leq y \leq 1, \quad 2y \leq z \leq 9 - 3xy^2.$$

What is the volume of $\mathcal{T}$?

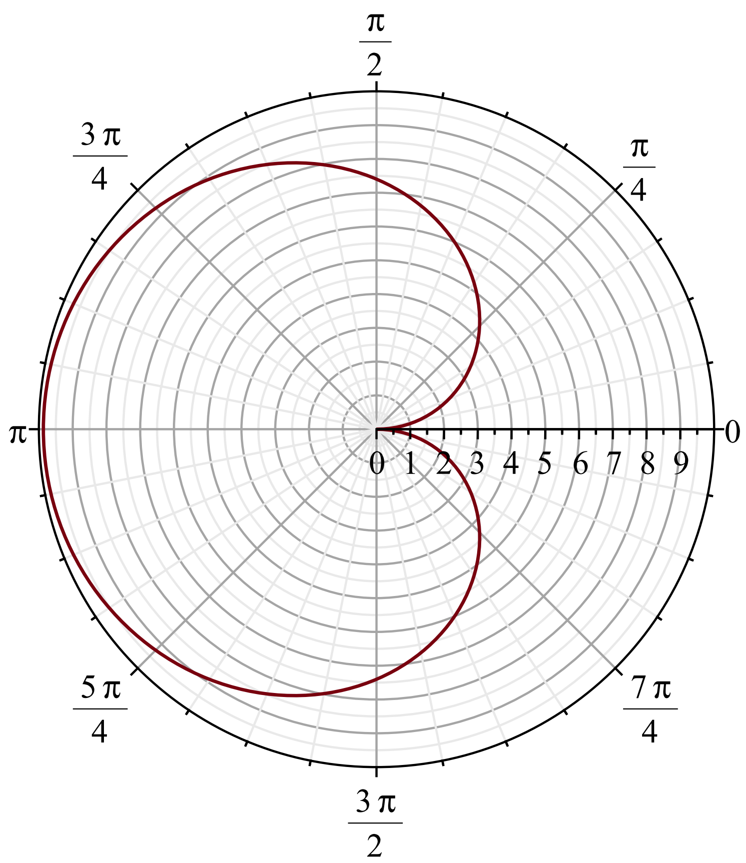
☐ 2	☐ $\frac{7}{2}$	☐ 12	☐ 16	☐ $18\sqrt{3}$
☐ 3	☐ 9	☑ 14	☐ 20	☐ $\frac{50}{3}$

Problem 12 (5 points)

The figure below shows the graph of the function

$$r = f(\theta), \quad 0 \leq \theta \leq 2\pi$$

in polar coordinates.



Which one of the following transformation rules for f corresponds to the figure?

- ☐ $f(\theta) = 1 + \cos\theta$
- ☐ $f(\theta) = 1 - \cos\theta$
- ☐ $f(\theta) = 1 - \theta^2$
- ☐ $f(\theta) = 8\sin^2\theta$
- ☑ $f(\theta) = \theta(2\pi - \theta)$
- ☐ $f(\theta) = 10 - \theta^2$