Reexam in Calculus

First Year at The Faculty of Engineering and Science
and The Faculty of Medicine

19 August 2016

The present exam set consists of 10 numbered pages with 14 multiple-choice problems. A number of points are specified for each question. The total number of points equals 100.

It is allowed to use books, notes etc. It **is not** allowed to use electronic devices.

Your answers must be marked directly in this exam set. The grading of the exam is based solely on these marks. Remember to write your **full name** and **student number** below.

Good luck!

NAME: ________________________________

STUDENT NUMBER: ________________________________

Problem 1 (6 points)

A curve in space is given by

$$\begin{aligned} x &= \cos(2t), \\ y &= \sin(2t), \\ z &= 2\ln(t), \end{aligned}$$

where the parameter t runs through the positive real numbers. Mark the correct expression for the arc length of the curve from $t = 1$ to $t = 2$.

☐ $\int_1^2 (\sin(2t) + \cos(2t) + t^{-1})\, dt$ ☐ $\int_1^2 4(1 + t^{-2})\, dt$

☐ $\int_1^2 2(\cos(2t) - \sin(2t) + t^{-1})\, dt$ ☐ $\int_1^2 2\sqrt{1 + t^{-2}}\, dt$

☐ $\int_1^2 2\sqrt{1 + t^{-1}}\, dt$ ☐ $\int_1^2 \sqrt{4 + t^{-1}}\, dt$

Problem 2 (8 points)

A plane curve is given by

$$\begin{aligned} x &= t^2 + t + 1, \\ y &= 2t^2 + t - 2, \end{aligned}$$

where the parameter t runs through the real numbers.

(a) (1 point). Which point on the curve corresponds to the parameter value $t = 0$?

☐ $(0, 0)$ ☐ $(1, -2)$ ☐ $(1, 1)$ ☐ $(1, 0)$ ☐ $(1, 3)$

(b) (7 points). What is the curvature of the curve for $t = 0$?

☐ $\frac{11}{4}$ ☐ 3 ☐ 1 ☐ $\frac{\sqrt{2}}{2}$ ☐ $\frac{1}{2}$

Problem 3 (7 points)

A function is defined by

$$f(x) = \frac{x}{x+1}.$$

(a) (3 points). Mark the correct expression for the double derivative $f''(x)$.

- ☐ $-\frac{6}{(x+1)^4}$
- ☐ $\frac{x^2}{(x+1)^3}$
- ☐ $-\frac{3x}{(x+1)^3}$
- ☐ $-\frac{2}{(x+1)^3}$
- ☐ 0
- ☐ $-\frac{3}{(x+1)^4}$

(b) (4 points). Which of the polynomials below is the 2nd order Taylor polynomial for $f(x)$ about the point $x = 0$?

- ☐ $x + 2x^2$
- ☐ $x - x^2$
- ☐ $1 + x^2$
- ☐ $1 + 3x^2$
- ☐ x
- ☐ $x + 3x^2$
- ☐ $2x + \frac{3}{2}x^2$
- ☐ $x - 3x^2$
- ☐ $x + x^2$
- ☐ $1 + 2x + x^2$

Problem 4 (5 points)

Evaluate the following integral and mark its value below.

$$\int_0^{\frac{1}{3}} \frac{3}{1+9t^2} dt.$$

- ☐ 1
- ☐ 3
- ☐ $\frac{\pi}{4}$
- ☐ -1
- ☐ $\frac{1}{2}$
- ☐ $\frac{\pi}{3}$

Problem 5 (5 points)

Consider the differential equation

$$y'' + 6y' + 9y = 0.$$

A number of function expressions, which contain two arbitrary constants c_1 and c_2, are listed below. Mark the expression which constitute the general solution of the differential equation.

☐ $y(t) = c_1 e^{2t}\cos(t) + c_2 e^{2t}\sin(t)$

☐ $y(t) = c_1 e^{6t} + c_2 e^{9t}$

☐ $y(t) = c_1 e^{2t} + c_2 t e^{2t}$

☐ $y(t) = c_1 e^{-2t} + c_2 e^{4t}$

☐ $y(t) = c_1 e^{3t}\cos(4t) + c_2 e^{3t}\sin(4t)$

☐ $y(t) = c_1 e^{-3t} + c_2 t e^{-3t}$

☐ $y(t) = c_1 e^{t}\cos(3t) + c_2 e^{t}\sin(3t)$

☐ $y(t) = c_1 e^{-t} + c_2 e^{-2t}$

☐ $y(t) = c_1 e^{4t}\cos(3t) + c_2 e^{4t}\sin(3t)$

☐ $y(t) = c_1 e^{-t} + c_2 t e^{-t}$

Problem 6 (8 points)

Consider the inhomogeneous differential equation

$$y'' + 3y' - y = 6e^t - 9e^{2t}.$$

(a) (3 points). Which one of the following functions $y_p(t)$ can be a particular solution of the differential equation for suitable values of the constants A and B?

- ☐ $y_p(t) = At + B$
- ☐ $y_p(t) = Ae^t + Be^{2t}$
- ☐ $y_p(t) = A\sin(t) + B\cos(2t)$
- ☐ $y_p(t) = Ae^{3t} + Be^{-t}$

(b) (5 points). What must the values of the constants A and B be in order to obtain a particular solution?

- ☐ $A = 2, \quad B = -1$
- ☐ $A = 3, \quad B = 1$
- ☐ $A = 1, \quad B = 3$
- ☐ $A = 1, \quad B = -1$

Problem 7 (9 points)

A function is given by

$$f(x, y) = y + \sqrt{2x - y}.$$

Mark the correct option in each of the subquestions below.

(a) (4 points). The domain for f consists of all points (x, y) which satisfy

- ☐ $x \geq y$
- ☐ $x \geq 0$ og $y \leq 0$
- ☐ $y \leq 2x$
- ☐ $y \neq 2x$
- ☐ $2x - y \leq 1$
- ☐ $y \geq 0$

(b) (5 points). The partial derivative $f_x(x, y)$ is equal to

- ☐ $1 + \frac{1}{2\sqrt{2x-y}}$
- ☐ $\frac{1}{\sqrt{2x-y}}$
- ☐ $\sqrt{2}$
- ☐ $1 + 4x$
- ☐ $\frac{1}{2\sqrt{2x-y}}$
- ☐ $\frac{1+y}{\sqrt{2x-y}}$

Problem 8 (10 points)

A function is defined by

$$f(x,y) = x^2 + 2y^2 + xy + x - 3y + 2.$$

(a) (2 points). Indicate the function value $f(1,1)$.

☐ 2 ☐ 4 ☐ 1 ☐ 0 ☐ 3

(b) (4 points). Which one of the following points is a critical point for f?

☐ $(1,1)$ ☐ $(1,3)$ ☐ $(-1,2)$ ☐ $(2,2)$ ☐ $(-1,1)$

(c) (4 points). The graph of f has a tangent plane at the point $P = (1,1,f(1,1))$. Mark an equation for this tangent plane.

☐ $x + 2y - 3z = 5$ ☐ $x - y + z = 4$

☐ $4x + y - z = 3$ ☐ $2x - z = 3$

☐ $5x + y + z = 11$ ☐ $4x + 2y - z = 2$

Problem 9 (8 points)

A function is given by

$$f(x,y) = e^x + y^2.$$

(a) (4 points). What is the value of the directional derivative $D_{\mathbf{u}}f(P)$ at the point $P = (0,1)$ and in the direction of the unit vector $\mathbf{u} = \frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j} = (\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$?

☐ $\sqrt{2}$ ☐ $\frac{1}{2}$ ☐ $-\frac{\sqrt{2}}{2}$ ☐ $\frac{9}{2}$ ☐ $\frac{3\sqrt{2}}{2}$

(b) (4 points). What is the maximal value of the directional derivative $D_{\mathbf{u}}f(P)$ at the point $P = (0,1)$ when $\mathbf{u}$ runs through all unit vectors?

☐ $\sqrt{5}$ ☐ 6 ☐ 9 ☐ $2\sqrt{3}$ ☐ 7

Problem 10 (10 points)

A region $\mathcal{R}$ in the plane consists of those points (x, y) which satisfy the inequalities

$$x^2 + y^2 \leq 1, \quad 0 \leq x, \quad 0 \leq y.$$

Mark the value of the double integral

$$\iint_{\mathcal{R}} (1 + \sqrt{x^2 + y^2})\, dA.$$

☐ $\frac{1}{3}$ ☐ $\frac{11}{3}$ ☐ $\sqrt{2\pi}$ ☐ $\frac{3}{2}$ ☐ $\frac{5\pi}{12}$

☐ $\frac{\pi}{4}$ ☐ 2π ☐ $\frac{\pi}{4}$ ☐ $\frac{1}{2}$ ☐ $\frac{\pi}{8}$

Problem 11 (5 points)

Two complex numbers are given by

$$z_1 = \frac{1 + 8i}{2 + i} + 1 - i, \qquad z_2 = e^{1 + \frac{\pi}{4} i} e^{2 + \frac{\pi}{4} i}.$$

(a) (3 points). What is z_1 written in standard form?

☐ $\frac{3}{2} + 7i$ ☐ $2 - 3i$ ☐ 7 ☐ $5 - 2i$ ☐ $3 + 2i$

(b) (2 points). What is z_2 written in standard form?

☐ e^3 ☐ $e^3 + e^3 i$ ☐ $e + i$ ☐ $e^3 i$ ☐ $3i$

Remark. In problem 12 the evaluation of your answers will be performed using the following principle: Each false mark cancels a true mark.

Problem 12 (6 points)

Let $\mathcal{T}$ be the region in space consisting of those points (x, y, z) which satisfy the inequalities

$$0 \leq x \leq 4, \quad 0 \leq y \leq \sqrt{x}, \quad -y \leq z \leq y.$$

A solid body with density function $\delta(x, y, z) = y + 1$ covers region $\mathcal{T}$ precisely. The volume of the body is denoted V and its mass is denoted m. Mark all of the correct expressions below.

☐ $m = \int_0^4 \int_0^{\sqrt{x}} \int_{-y}^{y} (y + 1)\, dzdxdy.$

☐ $m = \int_0^2 \int_{y^2}^{4} \int_{-y}^{y} (y + 1)\, dzdxdy.$

☐ $m = \int_0^4 \int_0^{\sqrt{x}} \int_{-y}^{y} (y + 1)\, dzdydx.$

☐ $V = \int_0^2 \int_0^{y^2} \int_{-y}^{y} dzdxdy.$

☐ $V = \int_0^4 \int_0^{\sqrt{x}} \int_{-y}^{y} dzdydx.$

Problem 13 (8 points).

Answer the following 4 True/False problems:

(a) (2 points). For the complex number $z = 1 + i$ one has $|z^4| = 4$.

☐ True ☐ False

(b) (2 points). Let D be the region in the plane consisting of those points (x, y) which satisfy the inequalities $0 \leq x \leq 1$ and $0 \leq y < 1$. Let f be the function with rule

$$f(x, y) = \frac{1 + x}{1 + x - y}$$

and domain D. Then f attains a global maximum on D.

☐ True ☐ False

(c) (2 points). For every real number x the following relation holds:

$$\sin(4x) = 4\sin(x).$$

☐ True ☐ False

(d) (2 points). The point with polar coordinates $(r, \theta) = (5, -\frac{\pi}{2})$ has rectangular coordinates

$$(x, y) = (0, -5).$$

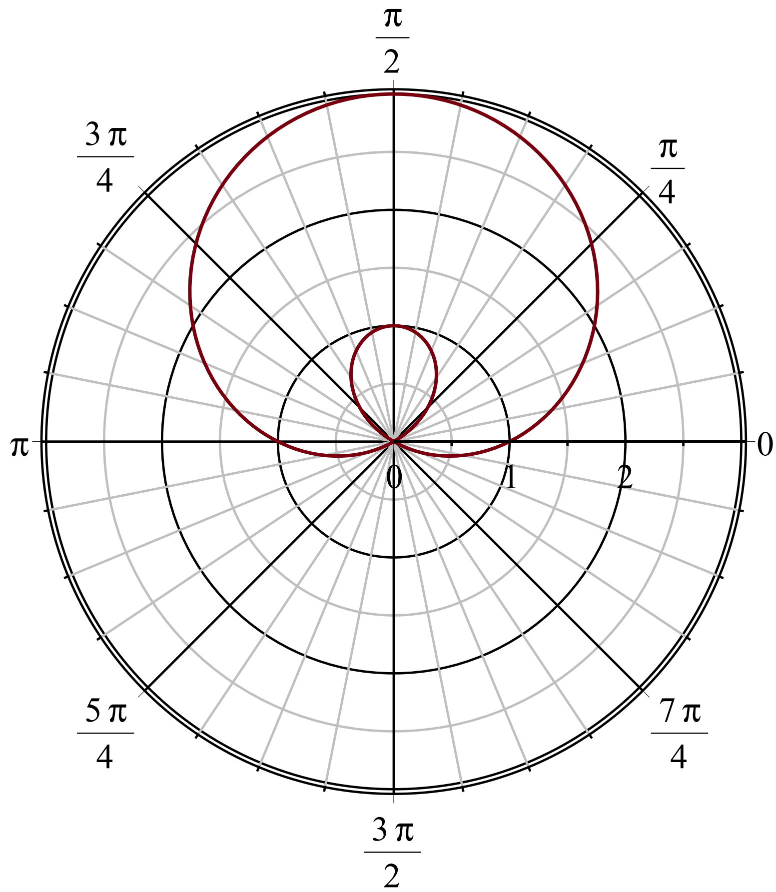
☐ True ☐ False

Opgave 14 (5 points)

The figure below shows the graph of the function

$$r = f(\theta), \quad 0 \le \theta \le 2\pi$$

in polar coordinates.



Which one of the following rules for f corresponds to the figure?

- ☐ $f(\theta) = 3 - 2\cos(\theta)$
- ☐ $f(\theta) = 1 + \cos(2\theta)$
- ☐ $f(\theta) = 2 + \sin(\theta)$
- ☐ $f(\theta) = 1 + 2\sin(\theta)$
- ☐ $f(\theta) = 2 - \cos(\theta)$
- ☐ $f(\theta) = 3\cos(\theta)$