Answers

Reexam in Calculus

**First Year at The Faculty of Engineering and Science
and The Faculty of Medicine**

19 August 2016

The present exam set consists of 10 numbered pages with 14 multiple-choice problems. A number of points are specified for each question. The total number of points equals 100.

It is allowed to use books, notes etc. It **is not** allowed to use electronic devices.

Your answers must be marked directly in this exam set. The grading of the exam is based solely on these marks. Remember to write your **full name** and **student number** below.

Good luck!

NAME: ______________________________

STUDENT NUMBER: ______________________________

Problem 1 (6 points)

A curve in space is given by

$$\begin{aligned} x &= \cos(2t), \\ y &= \sin(2t), \\ z &= 2\ln(t), \end{aligned}$$

where the parameter t runs through the positive real numbers. Mark the correct expression for the arc length of the curve from $t = 1$ to $t = 2$.

- ☐ $\int_1^2 (\sin(2t) + \cos(2t) + t^{-1})\,dt$
- ☐ $\int_1^2 4(1 + t^{-2})\,dt$
- ☐ $\int_1^2 2(\cos(2t) - \sin(2t) + t^{-1})\,dt$
- ☒ $\int_1^2 2\sqrt{1 + t^{-2}}\,dt$
- ☐ $\int_1^2 2\sqrt{1 + t^{-1}}\,dt$
- ☐ $\int_1^2 \sqrt{4 + t^{-1}}\,dt$

Problem 2 (8 points)

A plane curve is given by

$$\begin{aligned} x &= t^2 + t + 1, \\ y &= 2t^2 + t - 2, \end{aligned}$$

where the parameter t runs through the real numbers.

(a) (1 point). Which point on the curve corresponds to the parameter value $t = 0$?

☐ $(0,0)$ ☒ $(1,-2)$ ☐ $(1,1)$ ☐ $(1,0)$ ☐ $(1,3)$

(b) (7 points). What is the curvature of the curve for $t = 0$?

☐ $\frac{11}{4}$ ☐ 3 ☐ 1 ☒ $\frac{\sqrt{2}}{2}$ ☐ $\frac{1}{2}$

Problem 3 (7 points)

A function is defined by

$$f(x) = \frac{x}{x+1}.$$

(a) (3 points). Mark the correct expression for the double derivative $f''(x)$.

- ☐ $-\frac{6}{(x+1)^4}$
- ☐ $\frac{x^2}{(x+1)^3}$
- ☐ $-\frac{3x}{(x+1)^3}$
- ☒ $-\frac{2}{(x+1)^3}$
- ☐ 0
- ☐ $-\frac{3}{(x+1)^4}$

(b) (4 points). Which of the polynomials below is the 2nd order Taylor polynomial for $f(x)$ about the point $x = 0$?

- ☐ $x + 2x^2$
- ☒ $x - x^2$
- ☐ $1 + x^2$
- ☐ $1 + 3x^2$
- ☐ x
- ☐ $x + 3x^2$
- ☐ $2x + \frac{3}{2}x^2$
- ☐ $x - 3x^2$
- ☐ $x + x^2$
- ☐ $1 + 2x + x^2$

Problem 4 (5 points)

Evaluate the following integral and mark its value below.

$$\int_0^{\frac{1}{3}} \frac{3}{1+9t^2} dt.$$

- ☐ 1
- ☐ 3
- ☒ $\frac{\pi}{4}$
- ☐ -1
- ☐ $\frac{1}{2}$
- ☐ $\frac{\pi}{3}$

Problem 5 (5 points)

Consider the differential equation

$$y'' + 6y' + 9y = 0.$$

A number of function expressions, which contain two arbitrary constants c_1 and c_2, are listed below. Mark the expression which constitute the general solution of the differential equation.

- ☐ $y(t) = c_1e^{2t}\cos(t) + c_2e^{2t}\sin(t)$
- ☐ $y(t) = c_1e^{6t} + c_2e^{9t}$
- ☐ $y(t) = c_1e^{2t} + c_2te^{2t}$
- ☐ $y(t) = c_1e^{-2t} + c_2e^{4t}$
- ☐ $y(t) = c_1e^{3t}\cos(4t) + c_2e^{3t}\sin(4t)$
- ☒ $y(t) = c_1e^{-3t} + c_2te^{-3t}$
- ☐ $y(t) = c_1e^{t}\cos(3t) + c_2e^{t}\sin(3t)$
- ☐ $y(t) = c_1e^{-t} + c_2e^{-2t}$
- ☐ $y(t) = c_1e^{4t}\cos(3t) + c_2e^{4t}\sin(3t)$
- ☐ $y(t) = c_1e^{-t} + c_2te^{-t}$

Problem 6 (8 points)

Consider the inhomogeneous differential equation

$$y'' + 3y' - y = 6e^t - 9e^{2t}.$$

(a) (3 points). Which one of the following functions $y_p(t)$ can be a particular solution of the differential equation for suitable values of the constants A and B?

- ☐ $y_p(t) = At + B$
- ☒ $y_p(t) = Ae^t + Be^{2t}$
- ☐ $y_p(t) = A\sin(t) + B\cos(2t)$
- ☐ $y_p(t) = Ae^{3t} + Be^{-t}$

(b) (5 points). What must the values of the constants A and B be in order to obtain a particular solution?

- ☒ $A = 2, \quad B = -1$
- ☐ $A = 3, \quad B = 1$
- ☐ $A = 1, \quad B = 3$
- ☐ $A = 1, \quad B = -1$

Problem 7 (9 points)

A function is given by

$$f(x,y) = y + \sqrt{2x - y}.$$

Mark the correct option in each of the subquestions below.

(a) (4 points). The domain for f consists of all points (x,y) which satisfy

- ☐ $x \geq y$
- ☐ $x \geq 0$ og $y \leq 0$
- ☒ $y \leq 2x$
- ☐ $y \neq 2x$
- ☐ $2x - y \leq 1$
- ☐ $y \geq 0$

(b) (5 points). The partial derivative $f_x(x,y)$ is equal to

- ☐ $1 + \frac{1}{2\sqrt{2x-y}}$
- ☒ $\frac{1}{\sqrt{2x-y}}$
- ☐ $\sqrt{2}$
- ☐ $1 + 4x$
- ☐ $\frac{1}{2\sqrt{2x-y}}$
- ☐ $\frac{1+y}{\sqrt{2x-y}}$

Problem 8 (10 points)

A function is defined by

$$f(x,y) = x^2 + 2y^2 + xy + x - 3y + 2.$$

(a) (2 points). Indicate the function value $f(1,1)$.

☐ 2 ☒ 4 ☐ 1 ☐ 0 ☐ 3

(b) (4 points). Which one of the following points is a critical point for f?

☐ $(1,1)$ ☐ $(1,3)$ ☐ $(-1,2)$ ☐ $(2,2)$ ☒ $(-1,1)$

(c) (4 points). The graph of f has a tangent plane at the point $P = (1,1,f(1,1))$. Mark an equation for this tangent plane.

☐ $x + 2y - 3z = 5$ ☐ $x - y + z = 4$

☐ $4x + y - z = 3$ ☐ $2x - z = 3$

☐ $5x + y + z = 11$ ☒ $4x + 2y - z = 2$

Problem 9 (8 points)

A function is given by

$$f(x,y) = e^x + y^2.$$

(a) (4 points). What is the value of the directional derivative $D_{\mathbf{u}}f(P)$ at the point $P = (0,1)$ and in the direction of the unit vector $\mathbf{u} = \frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j} = (\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$?

☐ $\sqrt{2}$ ☐ $\frac{1}{2}$ ☒ $-\frac{\sqrt{2}}{2}$ ☐ $\frac{9}{2}$ ☐ $\frac{3\sqrt{2}}{2}$

(b) (4 points). What is the maximal value of the directional derivative $D_{\mathbf{u}}f(P)$ at the point $P = (0,1)$ when $\mathbf{u}$ runs through all unit vectors?

☒ $\sqrt{5}$ ☐ 6 ☐ 9 ☐ $2\sqrt{3}$ ☐ 7

Problem 10 (10 points)

A region $\mathcal{R}$ in the plane consists of those points (x, y) which satisfy the inequalities

$$x^2 + y^2 \leq 1, \quad 0 \leq x, \quad 0 \leq y.$$

Mark the value of the double integral

$$\iint_{\mathcal{R}} (1 + \sqrt{x^2 + y^2})\, dA.$$

☐ $\frac{1}{3}$ ☐ $\frac{11}{3}$ ☐ $\sqrt{2\pi}$ ☐ $\frac{3}{2}$ ☒ $\frac{5\pi}{12}$

☐ $\frac{\pi}{4}$ ☐ 2π ☐ $\frac{\pi}{4}$ ☐ $\frac{1}{2}$ ☐ $\frac{\pi}{8}$

Problem 11 (5 points)

Two complex numbers are given by

$$z_1 = \frac{1 + 8i}{2 + i} + 1 - i, \qquad z_2 = e^{1 + \frac{\pi}{4}i} e^{2 + \frac{\pi}{4}i}.$$

(a) (3 points). What is z_1 written in standard form?

☐ $\frac{3}{2} + 7i$ ☐ $2 - 3i$ ☐ 7 ☐ $5 - 2i$ ☒ $3 + 2i$

(b) (2 points). What is z_2 written in standard form?

☐ e^3 ☐ $e^3 + e^3 i$ ☐ $e + i$ ☒ $e^3 i$ ☐ $3i$

Remark. In problem 12 the evaluation of your answers will be performed using the following principle: Each false mark cancels a true mark.

Problem 12 (6 points)

Let $\mathcal{T}$ be the region in space consisting of those points (x, y, z) which satisfy the inequalities

$$0 \leq x \leq 4, \quad 0 \leq y \leq \sqrt{x}, \quad -y \leq z \leq y.$$

A solid body with density function $\delta(x, y, z) = y + 1$ covers region $\mathcal{T}$ precisely. The volume of the body is denoted V and its mass is denoted m. Mark all of the correct expressions below.

☐ $m = \int_0^4 \int_0^{\sqrt{x}} \int_{-y}^{y} (y+1)\, dzdxdy.$

☒ $m = \int_0^2 \int_{y^2}^{4} \int_{-y}^{y} (y+1)\, dzdxdy.$

☒ $m = \int_0^4 \int_0^{\sqrt{x}} \int_{-y}^{y} (y+1)\, dzdydx.$

☐ $V = \int_0^2 \int_0^{y^2} \int_{-y}^{y} dzdxdy.$

☒ $V = \int_0^4 \int_0^{\sqrt{x}} \int_{-y}^{y} dzdydx.$

Problem 13 (8 points).

Answer the following 4 True/False problems:

(a) (2 points). For the complex number $z = 1 + i$ one has $|z^4| = 4$.

☒ True ☐ False

(b) (2 points). Let D be the region in the plane consisting of those points (x, y) which satisfy the inequalities $0 \leq x \leq 1$ and $0 \leq y < 1$. Let f be the function with rule

$$f(x, y) = \frac{1+x}{1+x-y}$$

and domain D. Then f attains a global maximum on D.

☐ True ☒ False

(c) (2 points). For every real number x the following relation holds:

$$\sin(4x) = 4\sin(x).$$

☐ True ☒ False

(d) (2 points). The point with polar coordinates $(r, \theta) = (5, -\frac{\pi}{2})$ has rectangular coordinates

$$(x, y) = (0, -5).$$

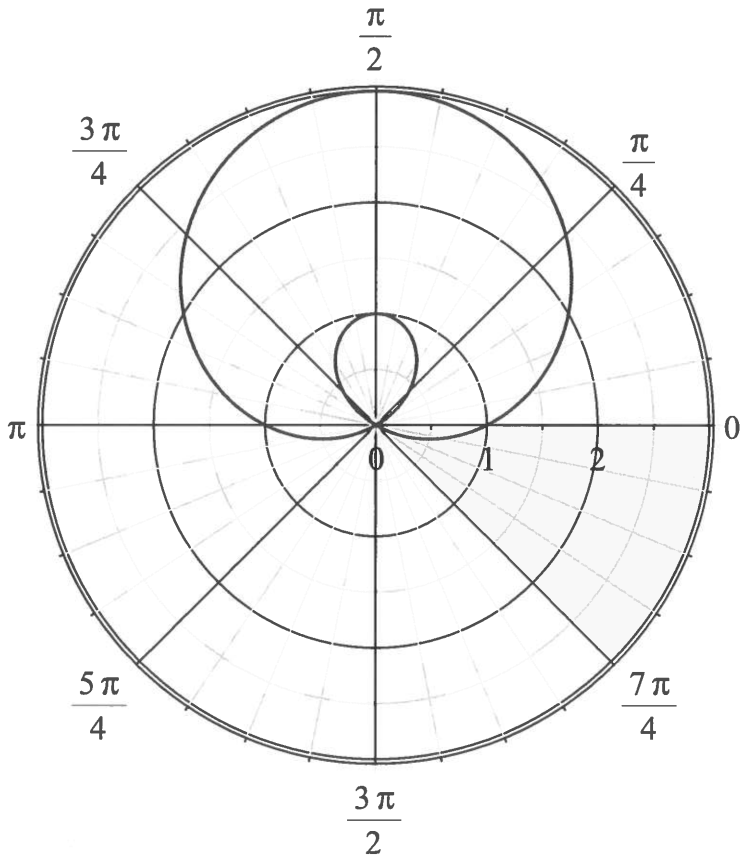
☒ True ☐ False

Opgave 14 (5 points)

The figure below shows the graph of the function

$$r = f(\theta), \quad 0 \le \theta \le 2\pi$$

in polar coordinates.



Which one of the following rules for f corresponds to the figure?

- ☐ $f(\theta) = 3 - 2\cos(\theta)$
- ☐ $f(\theta) = 2 + \sin(\theta)$
- ☐ $f(\theta) = 2 - \cos(\theta)$
- ☐ $f(\theta) = 1 + \cos(2\theta)$
- ☒ $f(\theta) = 1 + 2\sin(\theta)$
- ☐ $f(\theta) = 3\cos(\theta)$