Reexam in Calculus

First Year at the Faculty of Engineering and Science
and the Technical Faculty of IT and Design

17 February 2017

The present exam set consists of 11 numbered pages with 14 multiple-choice problems. A number of points are specified for each question. The total number of points equals 100.

It is allowed to use books, notes etc. It **is not** allowed to use electronic devices.

Your answers must be marked directly in this exam set. The grading of the exam is based solely on these marks. Remember to write your **full name** and **student number** below.

Good luck!

NAME: ______________________________

STUDENT NUMBER: ______________________________

Problem 1 (8 points)

Two complex numbers are given by

$$z_1 = 1 - 3i, \quad z_2 = 1 + 2i.$$

(a) (4 points). What is $z_1 z_2 - z_1$ written in standard form?

☐ $-3i$ ☐ $2+i$ ☐ $5-3i$ ☐ $6+2i$ ☐ $1-i$

(b) (4 points). What is $\overline{z}_1 - z_2$ written in polar form?

☐ $e^{\pi i/4}$ ☐ $3e^{\pi i/6}$ ☐ $\sqrt{2}\,e^{\pi i/4}$ ☐ $e^{-\pi i/4}$ ☐ $e^{\pi i/2}$

Problem 2 (6 points)

A homogeneous second order differential equation is given by

$$y'' - 8y' + 16y = 0.$$

A number of function expressions, which contain two arbitrary constants c_1 and c_2, are listed below. Mark the expression which constitute the general solution of the differential equation.

☐ $y(t) = c_1 e^{2t} + c_2 e^{8t}$

☐ $y(t) = c_1 e^{2t} + c_2 e^{4t}$

☐ $y(t) = c_1 e^{-2t} + c_2 e^{2t}$

☐ $y(t) = c_1 e^{4t} + c_2 t e^{4t}$

☐ $y(t) = c_1 e^{-t} + c_2 t e^{-t}$

☐ $y(t) = c_1 e^{-2t}\cos(4t) + c_2 e^{-2t}\sin(4t)$

☐ $y(t) = c_1 e^{-t}\cos(8t) + c_2 e^{-t}\sin(8t)$

☐ $y(t) = c_1 e^{2t}\cos(4t) + c_2 e^{2t}\sin(4t)$

Problem 3 (7 points)

The general solution of the differential equation

$$y'' - 6y' + 10y = 0$$

can be written as

$$y(t) = c_1 e^{3t} \cos t + c_2 e^{3t} \sin t,$$

where c_1 and c_2 are arbitrary constants.

(a) (2 points). Mark the correct expression for $y(0)$.

□ $c_1 + c_2$ □ c_1 □ $c_1 - c_2$ □ c_2 □ $3c_1 + 3c_2$

(b) (2 points). Mark the correct expression for $y'(0)$.

□ $3c_1$ □ $3c_1 + 3c_2$ □ $3c_2$ □ $3c_1 + c_2$ □ $c_1 - 3c_2$

(c) (3 points). The initial value problem

$$y'' - 6y' + 10y = 0, \quad y(0) = 2, \quad y'(0) = 1$$

has a unique solution $y(t)$. Find this solution and indicate the function value $y(\frac{\pi}{2})$ below.

□ $-5e^{3\pi/2}$ □ $5e^{3\pi/2}$ □ $2e^{3\pi/2}$ □ $-e^{\pi}$ □ $4e^{\pi/2}$

Problem 4 (7 points)

A function is defined by

$$f(x) = (1+x)\ln(1+x) - x$$

for $x > -1$.

(a) (3 points). What is the second order derivative $f''(x)$?

- ☐ $\frac{1}{1+x}$
- ☐ $\ln(x+1) - 1$
- ☐ $-\frac{1}{x}$
- ☐ $\frac{1}{(1+x)^3}$
- ☐ $\frac{1}{1+x} - 1$
- ☐ $\frac{1}{x} - 1$

(b) (4 points). Which one of the polynomials below is the third order Taylor polynomial for $f(x)$ about the point $x = 0$?

- ☐ $x^2 + \frac{3}{2}x^3$
- ☐ $x - \frac{1}{24}x^2$
- ☐ $1 - 5x$
- ☐ $\frac{1}{2}x^2 - \frac{1}{6}x^3$
- ☐ $\frac{1}{2}x^3$
- ☐ $x - \frac{1}{3}x^3$
- ☐ $2x^3$
- ☐ $x - x^2 + \frac{1}{6}x^3$
- ☐ $2 + x^2 - \frac{1}{2}x^3$
- ☐ $x^2 - x^3$

Problem 5 (9 points)

A curve in the plane is given by

$$x = e^t,$$
$$y = t^2,$$

where the parameter t runs through the real numbers.

(a) (3 points). Which one of the following points lies on the curve?

☐ $(e, 4)$	☐ $(1, 1)$	☐ $(2, \sqrt{2})$	☐ $(2, \frac{1}{2})$
☐ $(0, -1)$	☐ $(e^2, 4)$	☐ $(1, -1)$	☐ $(2, -\sqrt{2})$

(b) (6 points). The curvature of the curve is zero at a single point. Which one of the following points is it?

☐ $(1, 0)$	☐ $(0, 1)$	☐ $(e, 1)$	☐ $(2, (\ln 2)^2)$
☐ $(e^3, 9)$	☐ $(-1, 0)$	☐ $(e^{-2}, 4)$	☐ $(e^{-1}, 1)$

Problem 6 (8 points)

A curve in space is given by

$$x = 8t,$$
$$y = 3\sin(2t),$$
$$z = 3\cos(2t),$$

where the parameter t runs through the real numbers.

(a) (3 points). Mark the correct expression for the derivative z'.

☐ $-\sin(2t)$	☐ $6\cos(t)$	☐ $-6\sin(2t)$	☐ $-\sin(t)$
☐ $\sin(2t)$	☐ $-\sin(6t)$	☐ $\sin(5t)$	☐ $-\sin(5t)$

(b) (5 points). What is the arc length of the curve from $t = 1$ to $t = 2$?

☐ 5	☐ 10	☐ $\sqrt{6}$	☐ $2\sqrt{3}$
☐ 8	☐ 6	☐ $\frac{1}{2}$	☐ $\frac{5}{2}$

Problem 7 (8 points)

A function is given by

$$f(x,y) = \frac{2x+3}{x^2+y^2}.$$

Mark the correct option in each subquestion below.

(a) (4 points). The domain for f consists of all points (x,y) which satisfy

- ☐ $x^2 \geq y^2$
- ☐ $x > 0$ and $y > 0$
- ☐ $x > y$
- ☐ $x^2 + y^2 \geq 2x + 3$
- ☐ $2x \geq 3$
- ☐ $x \neq 0$ or $y \neq 0$

(b) (4 points). The level curve with equation $f(x,y) = 1$ can be described as:

- ☐ A parabola with equation $y = x^2 + x - 1$.
- ☐ A parabola with equation $y = x^2 - 2x + 1$.
- ☐ A straight line through $(-1, 0)$ with slope 2.
- ☐ A straight line through $(-1, 0)$ with slope -3.
- ☐ A circle with center $(1, 0)$ and radius 2.
- ☐ A circle with center $(2, 3)$ and radius $\frac{1}{2}$.

Problem 8 (6 points)

A function is defined by

$$f(x,y) = \arctan(x^2 - y^2).$$

(a) (3 points). What is the function value $f(\sqrt{2}, 1)$?

☐ 0 ☐ $\frac{\pi}{3}$ ☐ $\frac{\pi}{2}$ ☐ 1 ☐ $\frac{\pi}{4}$

(b) (3 points) What is the partial derivative $f_x(x,y)$?

☐ $(2x - 2y)(1 + (x^2 - y^2)^2)$ ☐ $\frac{2x-2y}{1+(x-y)^2}$

☐ $\frac{2x}{1+(x^2-y^2)^2}$ ☐ $2x(1 + (x^2 - y^2)^2)$

☐ $\frac{2xy}{1+(xy)^2}$ ☐ $\frac{2x}{1+x^2}$

Problem 9 (7 points)

A function is defined by

$$f(x,y) = x^2 + 3xy + 2y^2 + 2x + y + 1.$$

(a) (3 points). which one of the following points is a critical point for f?

☐ $(1,2)$ ☐ $(-1,3)$ ☐ $(3,3)$ ☐ $(4,-1)$ ☐ $(5,-4)$

(b) (4 points). What is the value of the directional derivative $D_{\mathbf{u}}f(P)$ at the point $P = (-2,1)$ and in the direction of the unit vector $\mathbf{u} = \frac{1}{2}\mathbf{i} - \frac{\sqrt{3}}{2}\mathbf{j} = (\frac{1}{2}, -\frac{\sqrt{3}}{2})$?

☐ 8 ☐ $\frac{\sqrt{3}+1}{2}$ ☐ $\sqrt{3}+2$ ☐ $2\sqrt{3}-1$ ☐ $\frac{\sqrt{3}-1}{4}$

Problem 10 (5 points)

A surface $\mathcal{F}$ in space is given by the equation $F(x, y, z) = 0$, where

$$F(x, y, z) = e^{x+y} - z + 2.$$

The surface $\mathcal{F}$ has a tangent plane at the point $P = (2, -2, 3)$. Mark an equation for this tangent plane below.

- ☐ $2y + z = -1$
- ☐ $x - y - z = 1$
- ☐ $2x + y - z = 1$
- ☐ $x + y + z = 3$
- ☐ $2x + 2y + z = 3$
- ☐ $-x + 2y + 2z = 0$
- ☐ $x + y - z = -3$
- ☐ $3x + 3y - 4z = -12$
- ☐ $-x + 2y + 4z = 2$
- ☐ $5x + 4y + z = 3$

Problem 11 (10 points)

A plane region $\mathcal{R}$ consists of those points (x, y) which satisfy the inequalities

$$0 \leq x \leq 1, \quad -x \leq y \leq \sqrt{x}.$$

Mark the value of the double integral

$$\iint_{\mathcal{R}} 2x^2 y \, dA.$$

☐ $\frac{1}{10}$	☐ $\frac{3}{10}$	☐ $\frac{1}{4}$	☐ 7	☐ $\frac{1}{3}$
☐ $\frac{5}{4}$	☐ $\frac{1}{20}$	☐ $\frac{11}{5}$	☐ $\frac{7}{2}$	☐ $\frac{3}{20}$

Remark. In Problem 12 the evaluation of your answers will be performed using the following principle: Each false mark cancels a true mark.

Problem 12 (6 points)

Let $\mathcal{T}$ be the region in space consisting of those points (x, y, z) which satisfy the inequalities

$$0 \leq x \leq 1, \quad -x \leq y \leq x, \quad 0 \leq z \leq 1 - x.$$

A solid body with density function $\delta(x, y, z) = 1 + x$ covers region $\mathcal{T}$ precisely. The volume of the body is denoted V and its mass is denoted m. Mark all of the correct expressions below.

☐ $m = \int_{-1}^{1} \int_{|y|}^{1} \int_{0}^{1-x} (1 + x)\, dzdxdy.$

☐ $m = \int_{0}^{1} \int_{-x}^{x} \int_{0}^{1-x} (1 + x)\, dzdydx.$

☐ $m = \int_{0}^{1} \int_{0}^{1-x} \int_{-x}^{x} (1 + x)\, dzdxdy.$

☐ $V = \int_{-1}^{1} \int_{y}^{1} \int_{0}^{1-x} dzdxdy.$

☐ $V = \int_{0}^{1} \int_{0}^{1} \int_{0}^{1-x} dzdydx.$

Problem 13 (8 points)

Answer the following 4 true/false problems:

(a) (2 points). The point with rectangular coordinates $(x, y) = (\sqrt{3}, 1)$ has polar coordinates $(r, \theta) = (1, \frac{\pi}{6})$.

☐ True ☐ False

(b) (2 points). For every complex number z one has

$$|\bar{z}|^4 = |z^4|.$$

☐ True ☐ False

(c) (2 points). Let D be the plane region consisting of those points (x, y) which satisfy the inequalities $0 \leq x \leq 1$ and $0 < y \leq 1$. Let f be the function with rule

$$f(x, y) = x + y$$

and domain D. Then f attains a global maximum on D.

☐ True ☐ False

(d) (2 points). The following equality holds:

$$\sqrt{2}\, e^{-i\pi/4} = 1 - i.$$

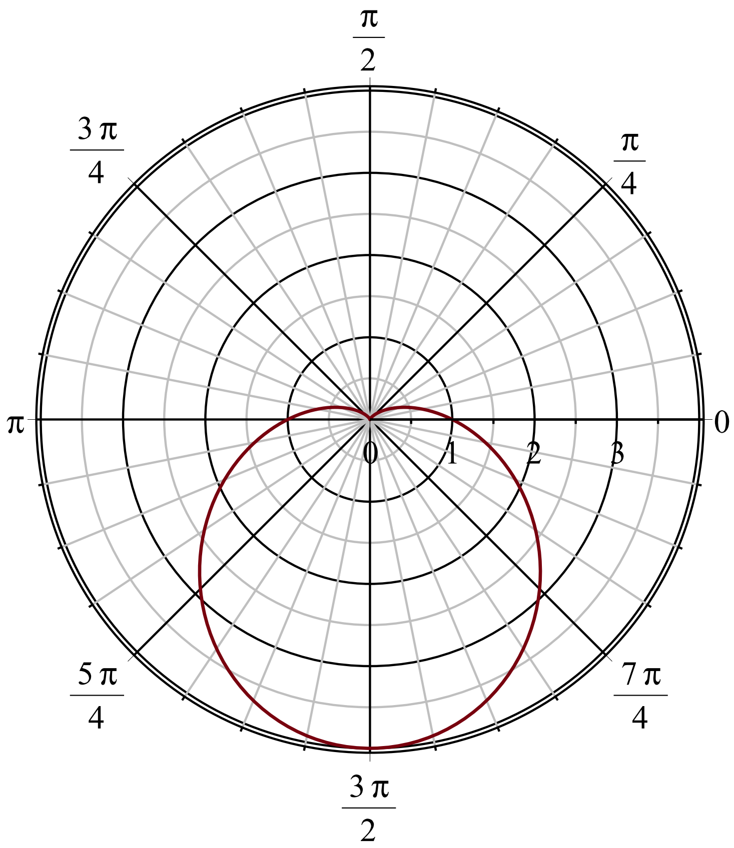
☐ True ☐ False

Problem 14 (5 points)

The figure below shows the graph of the function

$$r = f(\theta), \quad 0 \le \theta \le 2\pi$$

in polar coordinates.



Which one of the following rules for f corresponds to the figure?

- ☐ $f(\theta) = 3 + \cos^2\theta$
- ☐ $f(\theta) = 1 + \sin^2\theta$
- ☐ $f(\theta) = 1 - \sin\theta$
- ☐ $f(\theta) = 1 - \sin(2\theta)$
- ☐ $f(\theta) = (1 - \sin\theta)^2$
- ☐ $f(\theta) = 2 - \cos\theta$