

For at finde den danske version af prøven, begynd i den modsatte ende!

Please disregard the Danish version on the back if you participate in this English version of the exam.

Exam in Calculus

**First Year at the Technical Faculty for IT og Design,
the Faculty of Medicine and
the Faculty of Engineering and Science**

August 24, 2018

This test consists of 8 numbered pages and 12 multiple choice problems. A number of points are assigned to each problem. The entire test consists of 100 points in total.

It is allowed to use books, notes, etc. It is **not allowed** to use any **electronic devices**.

Your answers must be marked on these sheets. In each subproblem you should **only mark one of the listed choices**. The evaluation is solely based on your marked answers on these sheets.

Remember to write your **full name** and **student number** below. Moreover, please mark the team that you participate in.

Good luck!

NAME: _____

STUDENT NUMBER: _____

Team 2: EIT – ITC – PDP Henrik Garde

Team 3: ROB Anathanasios Georgiadis

with solutions

Problem 1 (6 points)

A function is given by

$$f(x, y) = \frac{x - 3y^2 - 1}{y^2 - x}$$

for real variables x and y .

(a) (2 points) The domain of f consists of all points (x, y) that satisfy

- | | | |
|--------------------------------------------------|-----------------------------------------|-----------------------------------------|
| <input type="checkbox"/> $y \neq \sqrt{ x }$ | <input type="checkbox"/> $x > 0, y < 0$ | <input type="checkbox"/> $3y^2 = x + 1$ |
| <input checked="" type="checkbox"/> $y^2 \neq x$ | <input type="checkbox"/> $x < 0, y > 0$ | <input type="checkbox"/> $y = \sqrt{x}$ |

(b) (4 points) Mark the correct expression for the level curve $f(x, y) = -3$.

- A parabola $x = 2y^2 + 1$
- A parabola $x = 3y^2 - 1$
- A circle with center $(-1, 0)$ and radius 1
- A circle with center $(-1, 0)$ and radius 1, excluding the point $(0, 0)$
- A straight line $x = -\frac{1}{2}$
- A straight line $x = -\frac{1}{2}$, excluding the points $(-\frac{1}{2}, -\frac{1}{\sqrt{2}})$ and $(-\frac{1}{2}, \frac{1}{\sqrt{2}})$

Problem 2 (6 points)

A parametrized space curve is given by

$$\mathbf{r}(t) = \langle e^t, e^{5t}, e^{(t^2)} \rangle$$

where the parameter t can take any real value.

(a) (3 points) What is the curve's speed at $t = 0$?

- | | | |
|---------------------------------------|-------------------------------------------------|--------------------------------------|
| <input type="checkbox"/> 0 | <input type="checkbox"/> $\sqrt{15}$ | <input type="checkbox"/> 5 |
| <input type="checkbox"/> $2\sqrt{13}$ | <input checked="" type="checkbox"/> $\sqrt{26}$ | <input type="checkbox"/> $\sqrt{30}$ |

(b) (3 points) Which of the following vectors agrees with the curve's acceleration vector at $t = 2$?

- | | | |
|----------------------------------------------------------------------------|---------------------------------------------------------------|-----------------------------------------------------------------|
| <input checked="" type="checkbox"/> $\langle e^2, 25e^{10}, 18e^4 \rangle$ | <input type="checkbox"/> $\langle e^2, e^{10}, e^4 \rangle$ | <input type="checkbox"/> $\langle 0, 0, 0 \rangle$ |
| <input type="checkbox"/> $\langle 1, 25, 10 \rangle$ | <input type="checkbox"/> $\langle e^2, 5e^{10}, 4e^4 \rangle$ | <input type="checkbox"/> $\langle e^2, 25e^{10}, 16e^4 \rangle$ |

Problem 3 (6 points)

Three complex numbers are given by

$$z_1 = 1 + 2i, \quad z_2 = 2e^{\frac{3\pi}{2}i} \quad \text{and} \quad z_3 = 2 + 2i.$$

(a) (3 points) What is $z_1 z_3$ on standard form?

- | | | |
|-----------------------------------|----------------------------------------------------------|-----------------------------------------------|
| <input type="checkbox"/> 0 | <input type="checkbox"/> $2 + 4i$ | <input type="checkbox"/> $1 + i$ |
| <input type="checkbox"/> $6 + 4i$ | <input type="checkbox"/> $2\sqrt{10}e^{\frac{2\pi}{3}i}$ | <input checked="" type="checkbox"/> $-2 + 6i$ |

(b) (3 points) What is $\frac{z_2}{z_3}$ on polar form?

- | | | |
|-----------------------------------------------------------|-------------------------------------------------|-----------------------------------------------------------------------------|
| <input type="checkbox"/> $\frac{1}{2}e^{\frac{3\pi}{2}i}$ | <input type="checkbox"/> 2 | <input checked="" type="checkbox"/> $\frac{1}{\sqrt{2}}e^{\frac{5\pi}{4}i}$ |
| <input type="checkbox"/> $4e^{\frac{5\pi}{4}i}$ | <input type="checkbox"/> $2e^{\frac{7\pi}{4}i}$ | <input type="checkbox"/> $e^{\frac{3\pi}{2}i}$ |

Problem 4 (10 points)

(a) (5 points) A homogeneous second order differential equation is given by

$$y'' + 2y = 0.$$

Several functions are given below, where c_1 and c_2 are arbitrary real constants. Mark the function which agrees with the general solution of the differential equation.

- | | |
|----------------------------------------------------------------------------------------|-----------------------------------------------------------------------|
| <input type="checkbox"/> $y(t) = c_1 e^{-\sqrt{2}t} + c_2 e^{\sqrt{2}t}$ | <input type="checkbox"/> $y(t) = c_1 e^t \cos(2t) + c_2 e^t \sin(2t)$ |
| <input checked="" type="checkbox"/> $y(t) = c_1 \cos(\sqrt{2}t) + c_2 \sin(\sqrt{2}t)$ | <input type="checkbox"/> $y(t) = c_1 \sin(2t) + c_2 \cos(2t)$ |
| <input type="checkbox"/> $y(t) = c_1 e^{-2t} + c_2 e^{2t}$ | <input type="checkbox"/> $y(t) = c_1 t^2 + c_2 \sqrt{2}t$ |
| <input type="checkbox"/> $y(t) = c_1 e^{-t} + c_2 t e^{-t}$ | <input type="checkbox"/> $y(t) = c_1 e^{-4t} + c_2 e^{2t}$ |

(b) (5 points) Mark a particular solution x_p of the inhomogeneous differential equation

$$x'' + 2x = 4t^2,$$

from the following list of functions.

- | | |
|--------------------------------------------------|-----------------------------------------------------------------|
| <input type="checkbox"/> $x_p(t) = 2t^2$ | <input type="checkbox"/> $x_p(t) = -4t^2 - t - 2$ |
| <input type="checkbox"/> $x_p(t) = -4t^2$ | <input checked="" type="checkbox"/> $x_p(t) = 2t^2 - 2$ |
| <input type="checkbox"/> $x_p(t) = t^2 + t + 1$ | <input type="checkbox"/> $x_p(t) = 3t^4 + 2t^2 + 2t - 1$ |
| <input type="checkbox"/> $x_p(t) = 2t^2 - t - 2$ | <input type="checkbox"/> $x_p(t) = \cos(t) + \sin(t) + t^2 - 2$ |

Problem 5 (8 points)

Mark whether the following statements regarding curvature are true or false.

(a) (2 points) A straight line can have positive curvature.

True

False

(b) (2 points) A circle with radius R has constant curvature $\frac{1}{R}$.

True

False

In subproblem (c) and (d) consider the following: Two particles $\mathbf{r}_A(t)$ and $\mathbf{r}_B(t)$ move along the same curve which contains a point P .

(c) (2 points) *The speed* of $\mathbf{r}_A(t)$ is twice as large as the speed of $\mathbf{r}_B(t)$. Then the curvature of $\mathbf{r}_A(t)$ at the point P is identical to the curvature of $\mathbf{r}_B(t)$ at P .

True

False

(d) (2 points) *The acceleration* of $\mathbf{r}_A(t)$ is twice as large as the acceleration of $\mathbf{r}_B(t)$. Then the curvature of $\mathbf{r}_A(t)$ at the point P is twice as large as the curvature of $\mathbf{r}_B(t)$ at P .

True

False

Problem 6 (6 points)

A function f is for $t \geq 0$ defined by

$$f(t) = e^t \cos(2t) + 4t^2 + t + 2.$$

Which of the following expressions agrees with $F(s) = \mathcal{L}(f)(s)$ for $s > 1$ (the Laplace transform of f)?

$\frac{2}{(s-1)^2+4} + \frac{8+s+2s^2}{s^3}$

$\frac{s-1}{(s-2)(s^2-1)} + \frac{8+2s^2}{s^3}$

$\frac{s-2}{(s-2)^2+4} + \frac{8}{s^3} + \frac{1+2s}{s^2}$

$\frac{2}{(s-1)^2+4} + \frac{4}{s^3} + \frac{1}{s^2} + \frac{2}{s}$

$\frac{s-1}{(s-1)^2+4} + \frac{8}{s^3} + \frac{1}{s^2} + \frac{2}{s}$

$\frac{s-2}{(s-2)+1} + \frac{8+s}{s^3} + \frac{2}{s}$

Problem 7 (6 points)

A function F is for $s > 3$ defined by

$$F(s) = \frac{6s - 2}{(s - 3)(s + 5)}.$$

Which of the following expressions agrees with $f(t) = \mathcal{L}^{-1}(F)(t)$ for $t \geq 0$ (the inverse Laplace transform of F)?

- | | | |
|------------------------------------------------|----------------------------------------------------------|-----------------------------------------------|
| <input type="checkbox"/> $e^{5t} + 2e^{-3t}$ | <input checked="" type="checkbox"/> $2e^{3t} + 4e^{-5t}$ | <input type="checkbox"/> $2e^{-3t} + 5e^{5t}$ |
| <input type="checkbox"/> $3te^{5t} + 3e^{-3t}$ | <input type="checkbox"/> $\frac{1}{2}e^{3t} + 4e^{-5t}$ | <input type="checkbox"/> $e^{-5t} + 2e^{3t}$ |

Problem 8 (10 points)

A surface \mathcal{F} is defined by the equation $F(x, y, z) = 0$, where

$$F(x, y, z) = -2 \sin(z) + yz - x^2y + y^2 - 1.$$

(a) (5 points) Which of the listed equations determines the tangent plane of \mathcal{F} at the point $P = (0, 1, 0)$?

- | | | |
|-----------------------------------------------------|--------------------------------------------------|--------------------------------------------|
| <input type="checkbox"/> $0 = x + \frac{3}{2}y + z$ | <input type="checkbox"/> $z = -\frac{1}{2}y - 1$ | <input type="checkbox"/> $z = 1$ |
| <input checked="" type="checkbox"/> $z = 2y - 2$ | <input type="checkbox"/> $2 = -2y + 2z$ | <input type="checkbox"/> $0 = -x - y + 3z$ |

(b) (5 points) From the equation $F(x, y, z) = 0$, what is the partial derivative $\partial z / \partial y$ evaluated at the point P ?

- | | | |
|-----------------------------------------|-----------------------------------------|--------------------------------|
| <input type="checkbox"/> -2 | <input type="checkbox"/> $-\frac{1}{2}$ | <input type="checkbox"/> 0 |
| <input checked="" type="checkbox"/> 2 | <input type="checkbox"/> 3 | <input type="checkbox"/> π |

Problem 9 (17 points)

A function is given by

$$f(x, y) = \arctan(2x + y) = \tan^{-1}(2x + y),$$

where the variables x and y can take any real value.

- (a) (4 points) Mark whether the following statement is true or false: The function f has at least one critical point.

True

False

- (b) (4 points) Mark whether the following statement is true or false: The function f has a global maximum.

True

False

- (c) (4 points) What is the directional derivative $D_{\mathbf{u}}f(P)$ at the point $P = (-1, 1)$ and in the direction given by the unit vector $\mathbf{u} = \langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$?

-2

$\frac{3}{\sqrt{2}}$

$\frac{3\sqrt{2}}{4}$

$\frac{1}{\sqrt{2}}$

$5\sqrt{2}$

4

- (d) (5 points) Which of the following unit vectors points in the direction of steepest ascend for f at the point P (the direction \mathbf{v} for which $D_{\mathbf{v}}f(P)$ is as large as possible)?

$\langle -\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \rangle$

$\langle \frac{2\sqrt{5}}{5}, -\frac{1}{\sqrt{5}} \rangle$

$\langle 0, -1 \rangle$

$\langle 0, 1 \rangle$

$\langle \frac{\sqrt{5}}{5}, \frac{2\sqrt{5}}{5} \rangle$

$\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$\langle \frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}} \rangle$

$\langle 1, 0 \rangle$

$\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \rangle$

Problem 10 (9 points)

A function is given by

$$f(x) = \ln(\sqrt{2x+1})$$

for $x > -2^{-1/2}$.

- (a) (5 points) Mark the expression which agrees with $f''(x)$ (*hint: remember to use the chain rule*).

$2 \ln(\sqrt{2x+1})$

$\frac{-2}{(\sqrt{2x+1})^2}$

$\frac{1}{\sqrt{2}}$

$\frac{\sqrt{2}}{\ln(\sqrt{2x+1})}$

$\frac{2}{2x^2+2\sqrt{2x+1}}$

$\frac{-2\ln(\sqrt{2x+1})}{(\sqrt{2x+1})^2}$

- (b) (4 points) Which of the following polynomials agrees with the second order Taylor polynomial of f about $x = 0$?

$1 + \sqrt{2}x^2$

$1 - x^2$

$\sqrt{2}x + \frac{1}{\sqrt{2}}x^2$

$1 + \sqrt{2}x + \sqrt{2}x^2$

$1 + 2x - x^2$

$\sqrt{2}x + 2x^2$

$-2x + \sqrt{2}x^2$

$\sqrt{2}x - x^2$

$2 - \sqrt{2}x - \sqrt{2}x^2$

Problem 11 (11 points)

A planar curve is given by

$$x = t + 3t^2,$$

$$y = 3t - t^2.$$

- (a) (2 points) For which value of the parameter t does the curve pass through the point $P = (4, 2)$?

$-\pi$

-1

$-\frac{\pi}{4}$

0

1

- (b) (4 points) What is the curvature of the curve at the point P ?

$\frac{\sqrt{2}}{25}$

$\frac{3}{50\sqrt{50}}$

0

$\frac{\sqrt{2}}{100}$

$\frac{20}{\sqrt{5}}$

- (c) (5 points) For which value of the parameter t is the curvature maximal?

-3

0

2π

-1

1

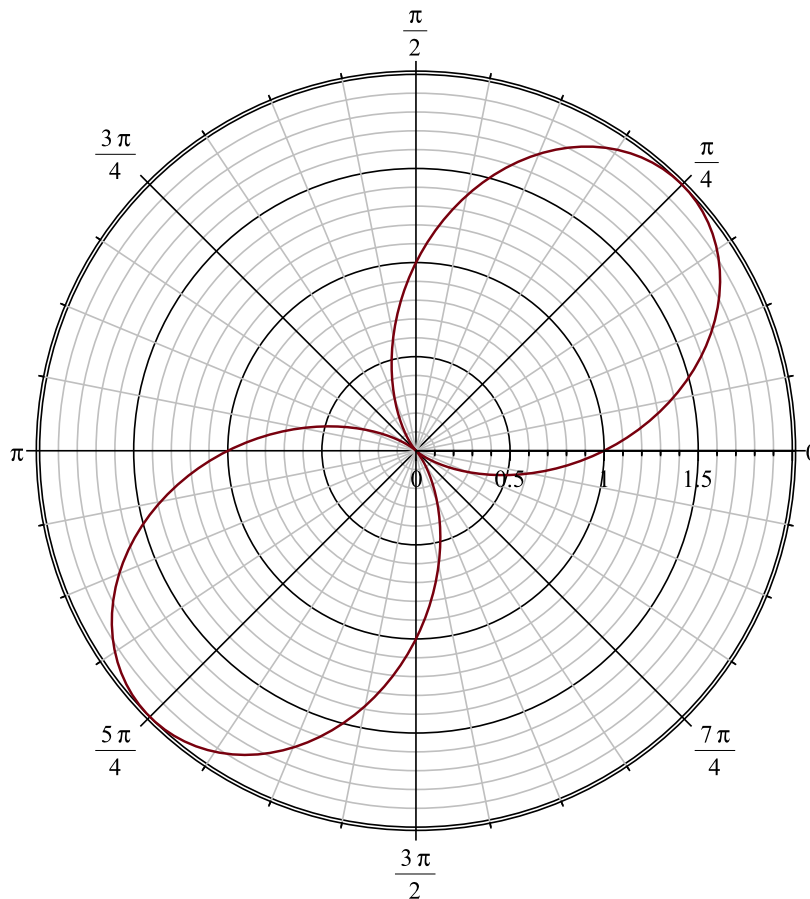
7

Problem 12 (5 points)

The figure below shows the graph of a function

$$r = f(\theta), \quad 0 \leq \theta \leq 2\pi,$$

in polar coordinates.



Which one of the functions below gives rise to that graph?

$f(\theta) = \sin(2\theta) + 1$

$f(\theta) = \theta^2 + 1$

$f(\theta) = \cos(4\theta) - 1$

$f(\theta) = \cos(2\theta) \sin(\theta)$

$f(\theta) = \sin(\theta) - \cos(\theta)$

$f(\theta) = 2 - \sin(2\theta)$