For at finde den danske version af prøven, begynd i den modsatte ende!

Please disregard the Danish version on the back if you participate in this English version of the exam.

Exam in Calculus

First Year at the Technical Faculty for IT and Design and the Faculty of Engineering and Science

June 12, 2017, 9:00 – 13:00

This test consists of 9 pages and 14 problems. All problems are "multiple choice" problems. Your answers must be given on these sheets.

It is allowed to use books, notes, xerox copies etc. It is **not allowed** to use any electronic devices such as pocket calculators, mobile phones or computers.

The listed percentages specify by which weight the individual problems influence the total examination.
Your marks will be evaluated as follows: For each sub-problem, every wrong mark will annul one correct mark.

Remember to write your full name (including middle names) together with your student number below. Moreover, please mark the team that you participate in.

Good luck!

NAME: ______________________________

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Problem 1 (8 points)

A planar curve is given by

$$x = e^t,$$
$$y = \ln t;$$

the parameter t can take any positive real value.

(a) (2 points) For which parameter t does the curve pass through the point $P = (e, 0)$?

☐ -1 ☐ 0 ☐ 1 ☐ e

(b) (2 points) Which of the following vectors is the velocity vector at P?

☐ $\begin{bmatrix} e \\ 1 \end{bmatrix}$ ☐ $\begin{bmatrix} e \\ 0 \end{bmatrix}$ ☐ $\begin{bmatrix} 0 \\ e \end{bmatrix}$ ☐ $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ ☐ $\begin{bmatrix} e \\ -e \end{bmatrix}$

(c) (4 points) Which of the following numbers agrees with the curvature $\kappa(P)$ at P?

☐ $\frac{2e}{e^2+1}$ ☐ $\frac{2e}{\sqrt{e^2+1}}$ ☐ $\frac{2e}{\sqrt{e^2+1}^3}$ ☐ $\frac{e}{\sqrt{e^2+1}^3}$ ☐ 0

Problem 2 (7 points)

The function f is defined by

$$f(x, y) = x^2 e^{-y}.$$

Via $z = f(x, y)$ it defines a surface containing the point $P = (1, -1, e)$.

(a) (4 points) Which of the following equations determine(s) the surface's tangent plane at P.

☐ $2e(x-1) - e(y+1) = z - e$ ☐ $z = 2x$

☐ $2(x-1) - (y+1) = z - e$ ☐ $z = e(2x - y - 2)$

☐ $2e(x-1) - e(y-1) = z + e$ ☐ $z = 2x - y - 2$

(b) (3 points) Which of the following expressions corresponds to the second order partial derivative $f_{xy}(x, y)$?

☐ $2e^{-y}$ ☐ $x^2 e^{-y}$ ☐ $-2xe^{-y}$ ☐ $-\frac{x^3}{3} e^{-y}$

Problem 3 (6 points)

A space curve is given by

$$x = \ln(t),$$
$$y = \sqrt{2}\,t,$$
$$z = \frac{1}{2}t^2;$$

the parameter t can take any positive real value

(a) (3 points) Mark the correct expression for the speed $v(t)$.

☐ $\sqrt{t^2+1}$ ☐ $\sqrt{\frac{t^2+1}{t}}$ ☐ $t+\frac{1}{t}$ ☐ $\sqrt{t^4+2t+1}$

(b) (3 points) Which of the following agrees with the arc length between $t = 1$ and $t = e$?

☐ $e+\frac{1}{e}-2$ ☐ $\frac{1}{2}(e^2+1)$ ☐ $\frac{e^2}{2}+1$ ☐ $e-\frac{1}{e^2}$ ☐ $\ln(2)+1$

Problem 4 (6 points)

A function is given by

$$f(x) = \sin(x^2)$$

for a real parameter x.

(a) (2 points) Which of the following is the derivative $f'(x)$?

☐ $\cos(x^2)$ ☐ $2x\sin(x^2)$ ☐ $\frac{x^3}{3}\cos(x^2)$

☐ $2x\cos(2x)$ ☐ $2x\cos(x^2)$ ☐ $-2x\cos(2x)$

(b) (4 points) Which of the following is the second degree Taylor polynomial of the function f at $x = 0$?

☐ x^2-x ☐ x^2 ☐ $1+x^2$ ☐ $2x+x^2$ ☐ $\frac{x^2}{2}$

Problem 5 (7 points)

A complex number has standard form $z = 1 + \sqrt{3}i$.

(a) (3 points) Which of the following agrees with $\bar{z}$ (z conjugate) in polar form?

☐ $2e^{-\frac{\pi i}{3}}$ ☐ $e^{\frac{\pi i}{4}}$ ☐ $\frac{1}{2}e^{-\frac{\pi i}{3}}$ ☐ $2e^{\frac{\pi i}{3}}$ ☐ $2e^{-\frac{\pi i}{6}}$

(b) (4 points) Which of the following agrees with z^3 in standard form?

☐ -2 ☐ 8 ☐ $1 - 3\sqrt{3}i$
☐ -8 ☐ $8i$ ☐ $-1 - \sqrt{3}i$

Problem 6 (9 points)

A complex number z on the imaginary axis is called *purely imaginary*, i.e., it has to be of the form $z = iy$ for some real number y.

(a) (3 points) For some complex numbers z, their square z^2 is purely imaginary.
Which of the following is a correct description of *all* these numbers?

☐ z purely imaginary ☐ $z = x(1 \pm i),\ x$ real
☐ all complex numbers ☐ $z = x(1 \pm i),\ x \geq 0$
☐ $z = x(1 + i),\ x$ real ☐ $z = x(1 - i),\ x$ real

(b) (3 points) For some complex numbers z, the product $z\bar{z}$ is a real number?
Which of the following is a correct description of *all* these numbers?

☐ all real numbers ☐ all complex numbers
☐ all purely imaginary numbers ☐ $z = x(1 + i), x$ real

(c) (3 points) For some complex numbers z, the quotient $\frac{\bar{z}}{z}$ is a real number?
Which of the following is a correct description of *all* these numbers?

☐ all numbers that are real or purely imaginary apart from 0 ☐ all complex numbers
☐ $z = x(1 - i), x$ real
☐ all real numbers ☐ $z = x(1 + i), x$ real
☐ all purely imaginary numbers

Problem 7 (9 points)

A second order homogeneous differential equation is given by

$$y'' - 4y' + 8y = 0.$$

(a) (3 points) The list below contains a number of function expressions including arbitrary constants c_1 and c_2. Mark the expression that describes all solutions of the differential equation.

☐ $y(t) = c_1 e^t \cos(2t) + c_2 e^t \sin(2t)$

☐ $y(t) = c_1 e^{2t} + c_2 e^{-2t}$

☐ $y(t) = c_1 e^{2t} + c_2 t e^{2t}$

☐ $y(t) = c_1 e^{2t} \cos(2t) + c_2 e^{2t} \sin(2t)$

☐ $y(t) = c_1 e^{-2t} \cos(2t) + c_2 e^{-2t} \sin(2t)$

☐ $y(t) = c_1 e^{2t} \cos(-2t) + c_2 e^{2t} \sin(-2t)$

☐ $y(t) = c_1 e^{t^2} \cos(t^2) + c_2 e^{t^2} \sin(t^2)$

(b) (3 points) Which of the following expressions is a solution of the differential equation with initial conditions $y(0) = 2, y'(0) = 6$?

☐ $e^{2t} + e^{-2t}$

☐ $e^{2t} - te^{-2t}$

☐ $e^{2t} \cos(2t) + 2e^{2t} \sin(2t)$

☐ $e^{2t} \cos(2t) + e^{2t} \sin(2t)$

☐ $2e^{-2t} \cos(2t) + e^{-2t} \sin(2t)$

☐ $e^{2t} \cos(-2t) + e^{2t} \sin(-2t)$

☐ $\frac{1}{2} e^{2t} \cos(2t) - \frac{1}{2} e^{2t} \sin(2t)$

☐ none of these

(c) (3 points) Which of the following function expressions is a (particular) solution of the inhomogeneous differential equation

$$y'' - 4y' + 8y = t^2?$$

☐ $t^3 + \frac{1}{2}t$

☐ $t^2 - 4t + 8$

☐ $\frac{t^2}{8} + \frac{t}{8} + \frac{1}{32}$

☐ $t + \frac{1}{8}$

☐ $\frac{t^2}{8} - \frac{t}{8} - \frac{1}{32}$

☐ $t^2 + t - \frac{1}{8}$

Problem 8 (9 points)

A function is given by the expression

$$z = f(x, y) = x^4 + y^4 - 4xy + 1$$

The graph of this function opens upward.

(a) (3 points) Which of the following are critical points for the function f?

☐ $(0,0)$ ☐ $(-1,1)$ ☐ $(1,1)$
☐ $(-1,-1)$ ☐ $(1,-1)$ ☐ $(2,3)$

(b) (3 points) Which of the following numbers is the minimal value of the function f?

☐ -3 ☐ -2 ☐ 1
☐ -1 ☐ 0 ☐ none of these

(c) (3 points) Does the function attain a local maximum at the point $(0,0)$?

☐ Yes ☐ No

Problem 9 (6 points)

A region $\mathcal{T}$ in 3D-space consists of all points (x, y, z) the coordinates of which satisfy the three inequalities

$$0 \leq x \leq 1, \quad -x^2 \leq y \leq x^2, \quad 0 \leq z \leq x^2 + y^2.$$

A body with density given by $\delta(x, y, z) = 2 - z$, with volume V and mass m covers the region $\mathcal{T}$.
Mark all correct expressions in the list below.

☐ $V = \int_0^1 \int_{-x^2}^{x^2} \int_0^{x^2+y^2} dzdydx.$

☐ $V = \int_0^1 \int_{\sqrt{|y|}}^{1} \int_0^{x^2+y^2} dzdxdy.$

☐ $V = \int_0^1 \int_{-x^2}^{x^2} \int_0^{x^2+y^2} dzdxdy.$

☐ $m = \int_0^1 \int_{-x^2}^{x^2} \int_0^{x^2+y^2} (2-z)dzdydx.$

☐ $m = \int_0^1 \int_{-x^2}^{x^2} \int_0^{x^2+y^2} (2-z)(x^2+y^2)dzdydx.$

Problem 10 (6 points)

A planar region $\mathcal{R}$ consists of all points (x, y) the coordinates of which satisfy the two inequalities

$$1 \leq x^2 + y^2 \leq 4, \quad 0 \leq y.$$

Mark the correct value of the planar integral

$$\iint_{\mathcal{R}} \frac{y}{\sqrt{x^2 + y^2}}\, dA.$$

☐ 6π	☐ $\frac{3}{2}$	☐ 1	☐ 3
☐ π	☐ 0	☐ 2	☐ $-\pi$

Problem 11 (7 points)

Consider the function

$$z = f(x, y) = (x + y)e^{x^2 - y^2}.$$

(a) (3 points) Which of the following numbers agrees with the directional derivative $D_{\mathbf{u}} f(P)$ at the point $P = (1, 1)$ in the direction determined by the unit vector
$\mathbf{u} = 0.8\mathbf{i} + 0.6\mathbf{j} = (0.8, 0.6)$?

☐ -0.6	☐ 0.6	☐ 5.8
☐ -0.4	☐ 0	☐ 2.2

(b) (4 points) Each of the following vectors $\mathbf{v}$ determines a unit vector $\mathbf{u}$ pointing in the same direction as $\mathbf{v}$. Mark all vectors in the list below for which the directional derivative $D_{\mathbf{u}} f(P) = 0$.

☐ $\mathbf{v} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$	☐ $\mathbf{v} = \begin{bmatrix} -3 \\ -5 \end{bmatrix}$	☐ $\mathbf{v} = \begin{bmatrix} 0.6 \\ 1 \end{bmatrix}$
☐ $\mathbf{v} = \begin{bmatrix} -0.6 \\ 0.8 \end{bmatrix}$	☐ $\mathbf{v} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$	☐ $\mathbf{v} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$

Problem 12 (7 points)

A function is given by the expression

$$f(x,y) = \frac{x^2}{x^2+y^2}.$$

(a) (2 points) Mark whether the domain of the function f consists of all points satisfying

- □ $x \neq -y$
- □ $x \geq 0$ og $y \geq 0$
- □ $x \neq 0$ and $y \neq 0$
- □ $x = y = 0$
- □ $x \neq 0$ or $y \neq 0$
- □ $x \neq 0$

(b) (3 points) Which of the following descriptions fits with the level curve

$$f(x,y) = \frac{1}{4}?$$

- □ A circle with center at $(1,0)$ and radius 1.
- □ A line through $(0,0)$ with slope $\sqrt{3}$.
- □ Two lines through $(0,0)$ with slope $\pm\sqrt{3}$.
- □ Two lines through $(0,0)$ with slope $\pm\sqrt{3}$ – apart from the origin.
- □ A line through $(0,0)$ with slope $\sqrt{3}$ – apart from the origin.
- □ Two lines through $(0,0)$ with slope $\pm\frac{1}{\sqrt{3}}$ – apart from the origin.

(c) (2 points) Which of the following descriptions fits with the level curve

$$f(x,y) = 4?$$

- □ an ellipse
- □ a hyperbola
- □ two straight lines
- □ the origin $(0,0)$
- □ the empty set
- □ a parabola

Problem 13 (6 points)

A surface $\mathcal{F}$ in 3D space is given implicitly by the following equation

$$F(x, y, z) = 2xy + 3xz - yz = 2.$$

Which of the following equations below are descriptions of the tangent plane to the surface $\mathcal{F}$ at the point $P = (1, -1, 1)$?

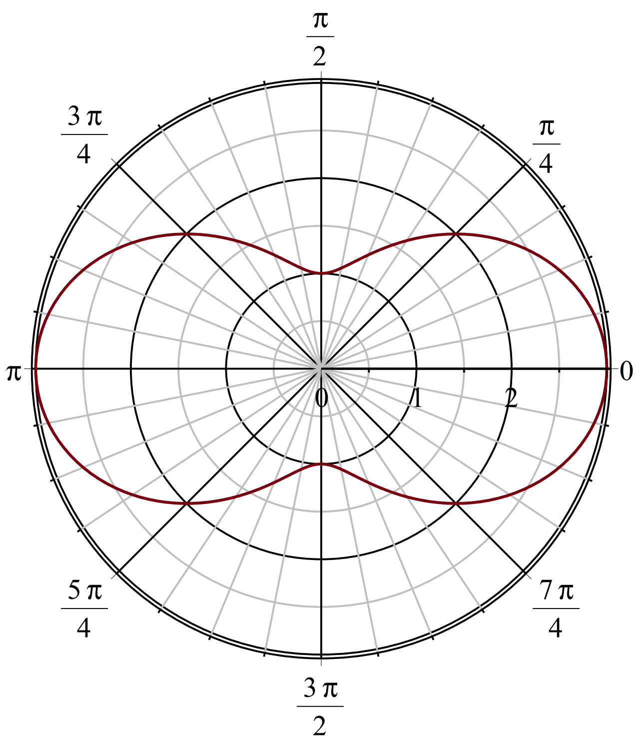
- ☐ $2x + 3y - z = 2$
- ☐ $-x - y - 4z = -4$
- ☐ $2z + 3y - x = 2$
- ☐ $x - y + z = 3$
- ☐ $x + y + 4z = 4$
- ☐ $2x + 2y + 8z = 8$
- ☐ $x - y + 4z = 6$
- ☐ $x + y + 4z = 8$

Problem 14 (7 points)

The figure below shows the graph of a function

$$r = f(\theta), \quad 0 \leq \theta \leq 2\pi.$$

in polar coordinates.



Which of the functions below gives rise to that graph?

- ☐ $f(\theta) = 1 + \cos(\theta)$
- ☐ $f(\theta) = 2 + \sin(2\theta)$
- ☐ $f(\theta) = 2 + \sin(\theta)$
- ☐ $f(\theta) = 2 + \cos(2\theta)$
- ☐ $f(\theta) = 2 + \cos(\theta)$
- ☐ $f(\theta) = (1 - \sin(\theta))^2$