Re-Exam 2011

Mathematics for Multimedia Applikations
AAU-Cph, Medialogy

11. August 2011

Formalities

This exam set consists of 8 pages. There are 10 problems containing 33 sub-problems in total. Books and notes are allowed but *no electronic devices* such as calculators, computers or cell phones are permitted.

A number of points is indicated for every sub-problem. The sum of these points equals 100.

Date and time for the exam: 11. August, 9:00 - 13:00

You must indicate the following on each page:

- Full name

- Study number

- Page number

On the first page, you must indicate

- The total number of pages.

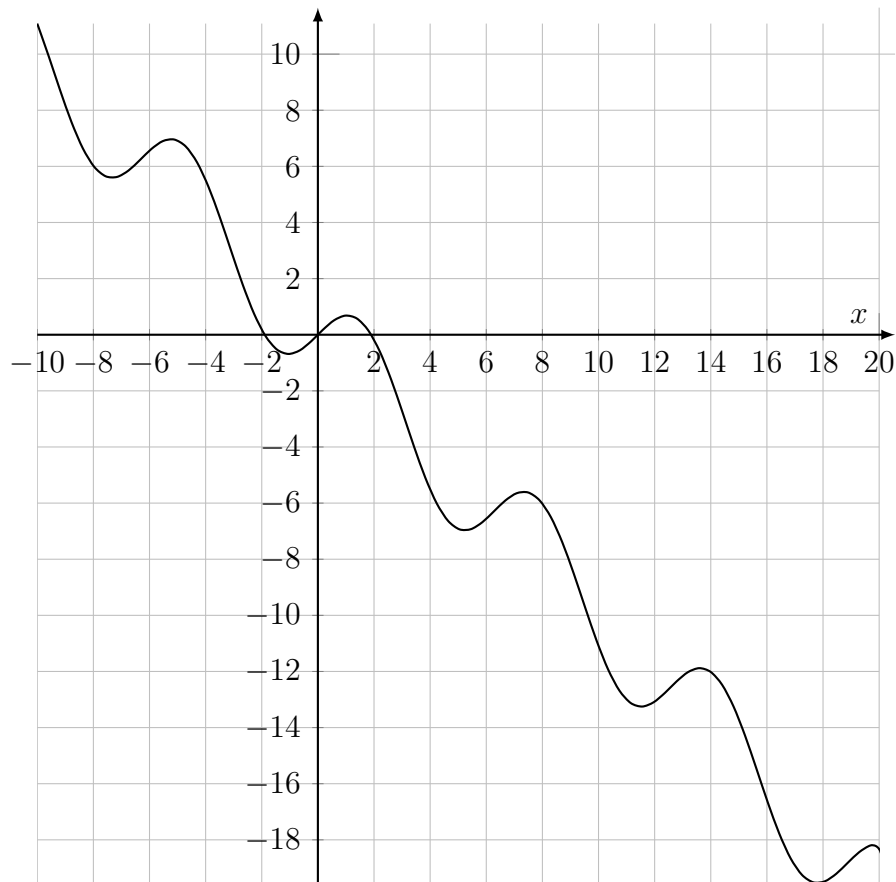 Remark that special values for sine and cosine are added as an appendix.

Good luck!

Problems

Problem 1.

1.a. (3 points) Find the derivative of the function $x + x^2 \tan(x)$ with respect to x.

1.b. (3 points) Let $f(x) = \sqrt{\cos(x) + x^2}$. Calculate $f'(x)$.

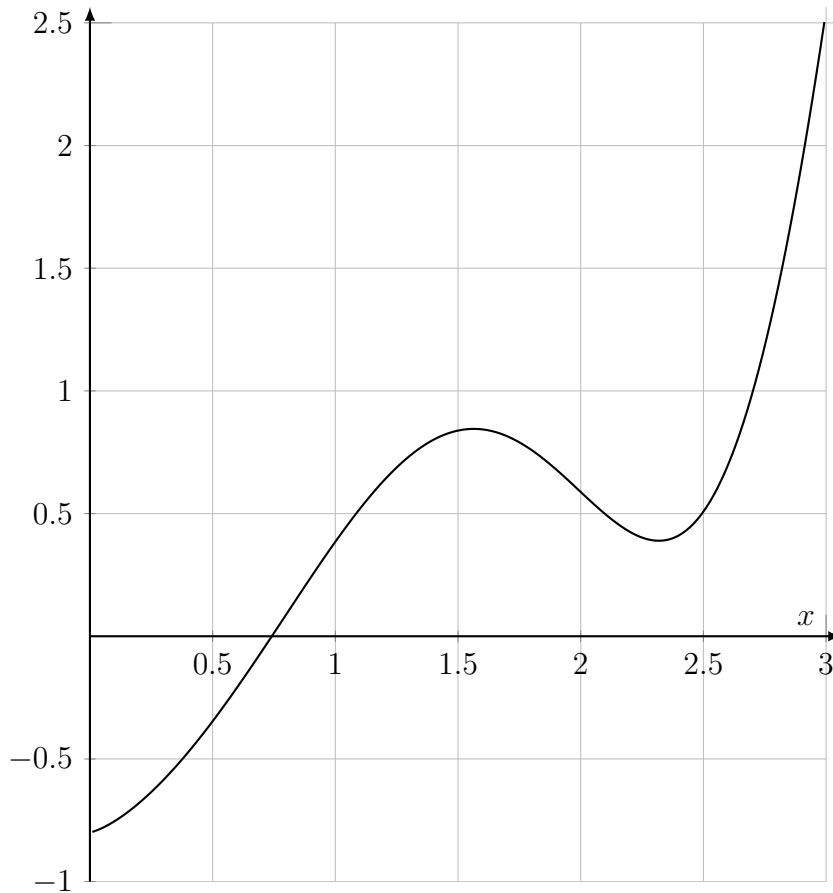**Problem 2.** Let $f(x) = 2\sin(x) - x$. The graph of this function looks as follows:

2.a. (2 points) Compute $f'(x)$.

2.b. (3 points) Find an x such that $f'(x) = 0$.

2.c. (5 points) Describe all x such that $f'(x) = 0$.

Problem 3. Consider the graph of a function f:

Use the sheet on page 8 for your answers to the following:

3.a. (2 points) Indicate all points where $f'(x) = 0$.

3.b. (2 points) Sketch the tangents to the graph for f at $(\frac{3}{4}, f(\frac{3}{4}))$ and at $(2, f(2))$.

3.c. (2 points) Mark by intervals on the x-axis where the derivative is positive and where it is negative.

3.d. (4 points) Sketch the graph of the derivative f'.

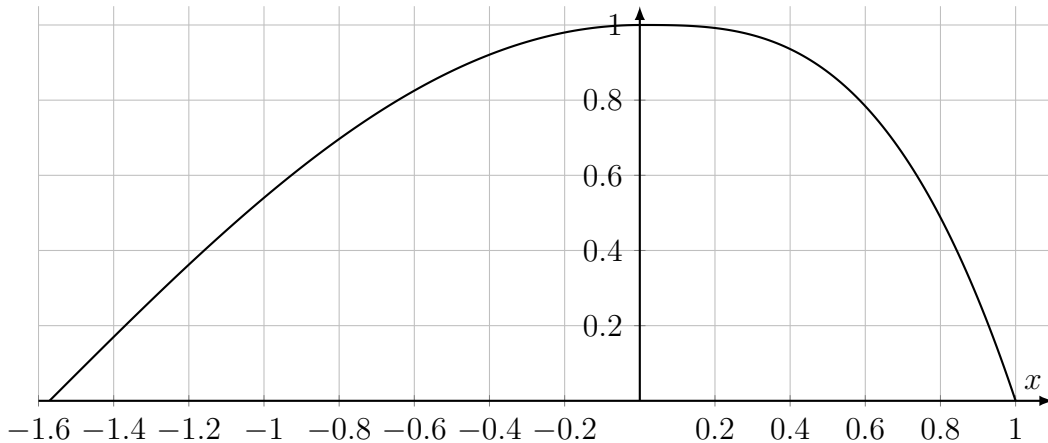**Problem 4.** Let $g(x) = \log_e(3x^2)$, where e is the base of the natural exponential function. Let
$$f(x) = xe^{g(x)} - 2x^3.$$

4.a. (3 points) Find a reduced form of the function f which does not depend on the function g.

Problem 5. Let f be the function defined by

$$f(x) = \begin{cases} \cos(x), & x \in [-\frac{\pi}{2}, 0], \\ 1 - x^3, & x \in [0, 1]. \end{cases}$$

The graph of f looks as follows:

5.a. (3 points) Compute $\int_{-\pi/2}^{0} f(x)\, dx$.

5.b. (3 points) Compute $\int_{0}^{1} f(x)\, dx$.

5.c. (3 points) Find $\int_{-\pi/2}^{1} f(x)\, dx$.

Problem 6. Let P, Q and R be three points in 3D-space; P has coordinates $(2, 3, 1)$, Q has coordinates $(2, 4, 2)$ and R has coordinates $(1, 3, 2)$.

6.a. (2 points) Find $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

6.b. (3 points) Compute the cross product $\overrightarrow{PQ} \times \overrightarrow{PR}$.

6.c. (2 points) Compute the dot product $\overrightarrow{PQ} \bullet \overrightarrow{PR}$.
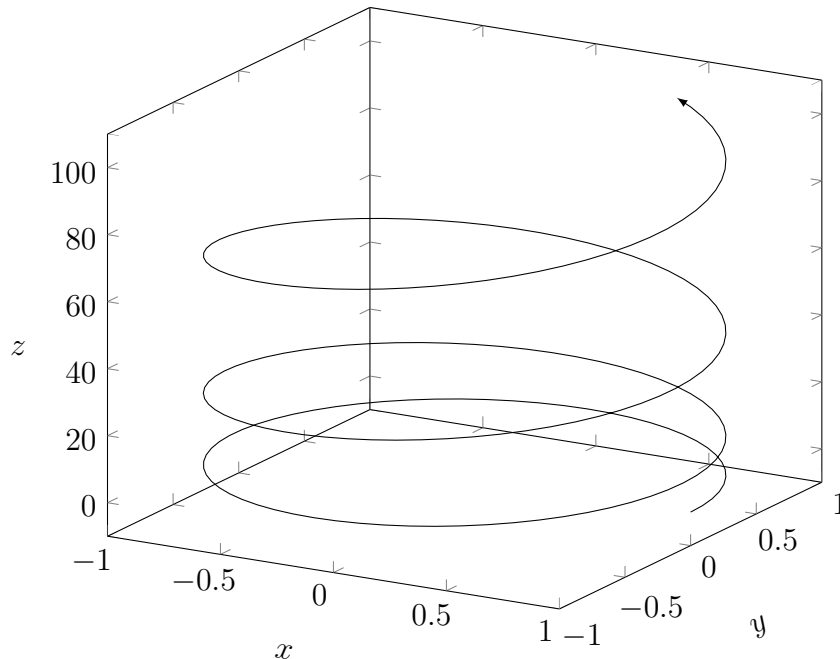
6.d. (3 points) Find the angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

6.e. (3 points) Find the area of the triangle with vertices P, Q and R.

Problem 7. A parametric curve is given by the following vector function:

$$\vec{r}(t) = (\cos(2t), \sin(2t), t^2)$$

Here is a plot of the curve when t runs from 0 to 10:

7.a. (2 points) Compute the velocity vector $\vec{v}(t)$.

7.b. (3 points) Compute the speed $\nu(t)$.

7.c. (3 points) Find a t such that the speed equals 2.

Problem 8. Consider the following system of linear equations:

$$x_1 + x_2 + 7x_3 = 2$$
$$3x_1 + x_2 + 11x_3 = 8$$
$$x_1 + 2x_2 + 12x_3 = 1$$

8.a. (2 points) Find the augmented matrix of the system.

8.b. (4 points) Find a row echelon form of the augmented matrix.

8.c. (3 points) Find the reduced row echelon form of the augmented matrix.

8.d. (4 points) Write down the general solution to the system.

8.e. (3 points) One solution to the system is $x_1 = 3$, $x_2 = -1$, $x_3 = 0$. Find another solution which has $x_3 = 1$.

8.f. (3 points) Find a solution to the system which has $x_2 = 9$.

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Problem 9.

9.a. (7 points) Find a polynomial function $f(x) = ax^2 + bx + c$ whose graph passes
through the points $(1, 6)$, $(-1, -4)$ and $(2, 8)$.

Problem 10. Define two matrices as follows:
$$A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} -7 & 1 \\ 1 & 0 \end{bmatrix}$$

10.a. (2 points) Compute the matrix product AB.

10.b. (2 points) Let $T(\vec{x}) = A\vec{x}$. Compute $T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right)$.

10.c. (3 points) Find an $\vec{x}$ such that $T(\vec{x}) = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$.

10.d. (5 points) Show that A and B are invertible and compute A^{-1} and B^{-1}.

10.e. (3 points) Find $(AB)^{-1} - B^{-1}A^{-1}$.

Appendix

Exact values of sin and cos for some angles:

- $\sin(\pi/6) = \cos(\pi/3) = 1/2$
- $\sin(\pi/4) = \cos(\pi/4) = 1/\sqrt{2} = \sqrt{2}/2$
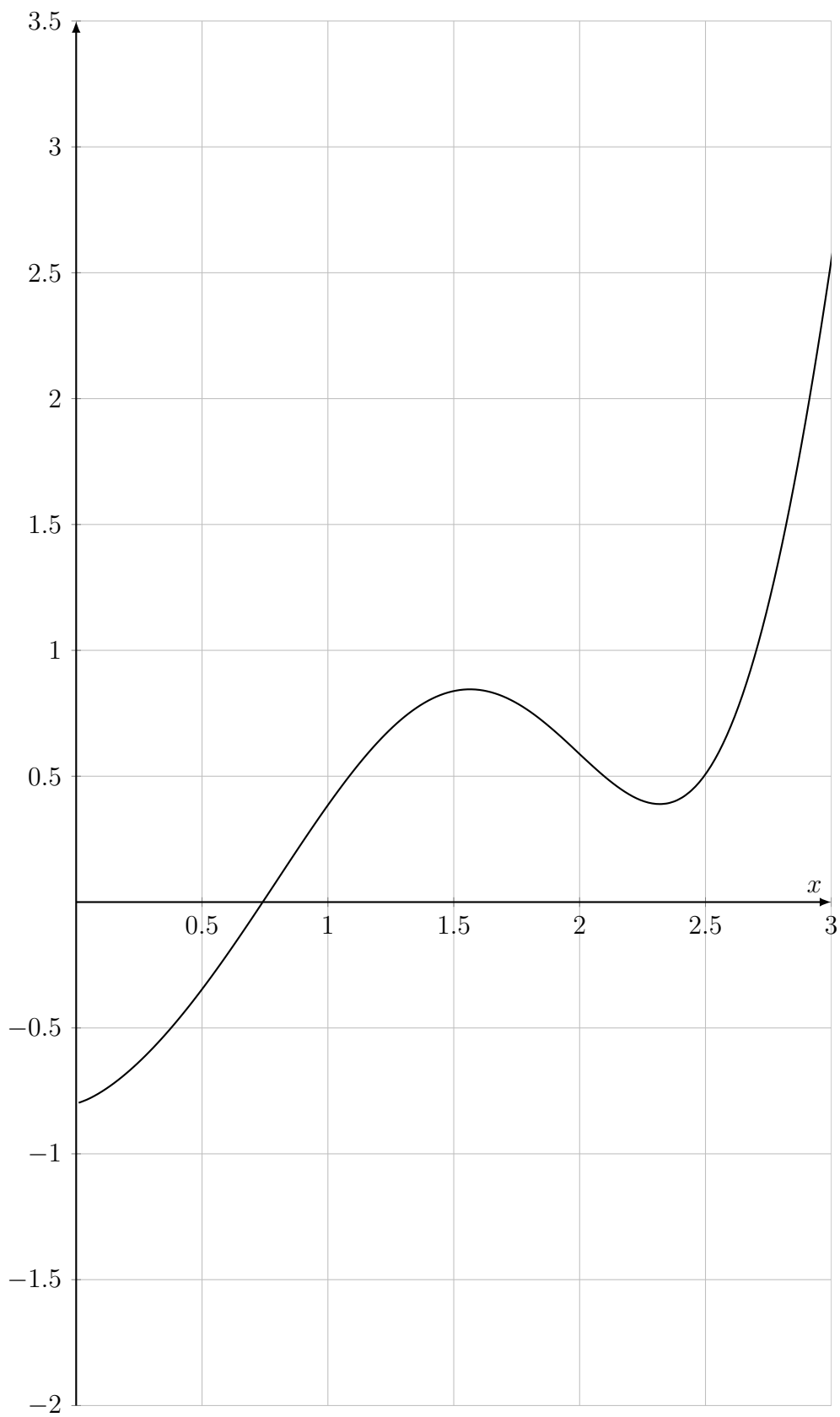- $\sin(\pi/3) = \cos(\pi/6) = \sqrt{3}/2$

Figure 1: The function f from Problem 3.