Self-study session 2, Calculus

First year mathematics for the technology and science programmes
Aalborg University

The purpose of this self-study session is to provide a perspective on some of the key concepts and approaches presented in E&P Chapter 12. Feel free to use Matlab or Maple\textsuperscript{1} for symbolic computations.

Part I: Tangent plane and optimization

A function is for all \((x, y) \in \mathbb{R}^2\) defined by

\[
f(x, y) = 4xy^2 + 2x^2 + 6y^2 + 10.
\]

The surface \(F\) is the graph for \(f(x, y)\). That is, \(F\) is given by the equation \(z = f(x, y)\).

a) Determine the equation for the tangent plane of the surface \(F\) at the point \(P(-1, 2, f(-1, 2))\).

b) Determine those points \((x, y, f(x, y))\) at which the tangent planes of \(F\) are parallel with the \(xy\)-plane.

c) A region \(R\) is determined by \(y \geq 0, x \leq 0\) and \(y^2 \leq x + 6\). Sketch the region \(R\).

d) Determine the maximal and minimal values of \(f(x, y)\) in the region \(R\).

Part II: Gradient vector, chain rule and implicitly defined functions

A function \(F\) is defined by

\[
F(x, y, z) = x^2 \cos y + 2y \cos x + 3z - \sin z.
\]

a) Determine the gradient vector \(\nabla F\) to \(F\) at the point \(P(0, 0, 0)\).

b) Determine the directional derivative to \(F\) at the point \(P\) in the direction given by the vector \(v = [2, 2, -1]^T\).

c) The function \(f(x, y)\) is defined implicitly by \(F(x, y, f(x, y)) = 0\). Determine \(f(0, 0), f_x(0, 0)\) and \(f_y(0, 0)\).

\textsuperscript{1}Partial derivatives can easily be calculated symbolically in Matlab and Maple. As an example here is how to compute \(f_x\) for the function \(f(x, y) = x^2 y^3\):

\[
\begin{align*}
&\text{syms x y} \\
&f = x^2 y^3 \\
&\text{diff}(f, x)
\end{align*}
\]

In Maple use: `diff(x^2*y^3, x)`.