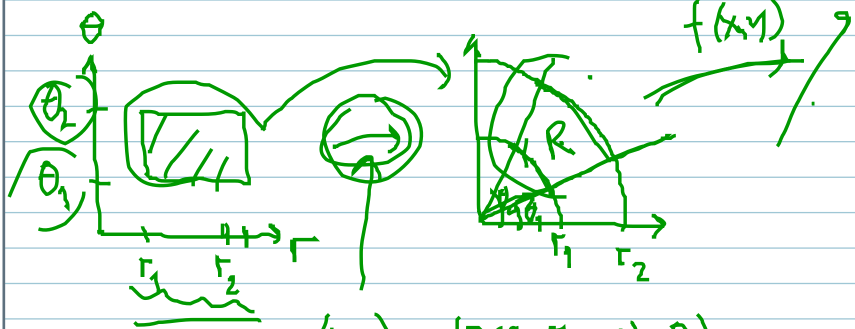
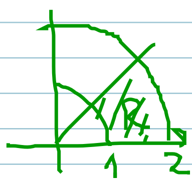
Integration over polære rektangler



$$(x,y) = (r\cos\theta, r\sin\theta)$$

$$\iint_R f(x,y)\,dA = \int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$



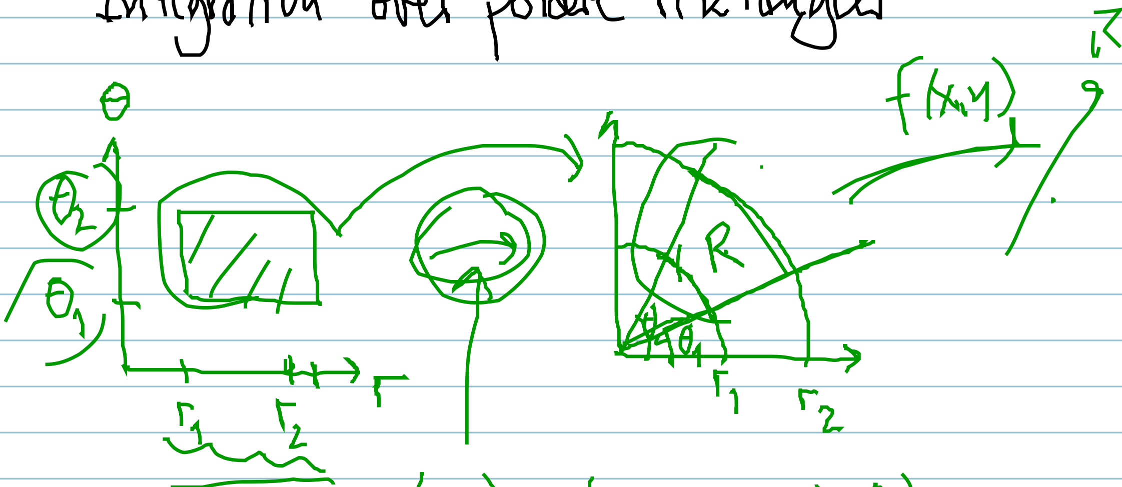
$$0 \le \theta \le \frac{\pi}{4} \quad 1 \le r \le 2$$

$$f(x,y) = e^{-x^2} \cdot e^{-y^2} = e^{-(x^2+y^2)}$$

$$\iint_R f(x,y)\,dA = \int_0^{\frac{\pi}{4}} \int_1^2 e^{-r^2} \cdot r\, dr\, d\theta$$

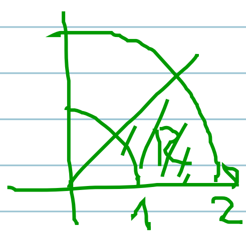
$$\frac{d}{dr} e^{-r^2} = e^{-r^2} \cdot (-2r)$$

Integration over polare rektangler



$$(x,y) = \underline{(r\cos\theta,\ r\sin\theta)}$$

$$\iint_R f(x,y)\,dA = \int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$



$$0 \le \theta \le \frac{\pi}{4} \qquad 1 \le r \le 2$$

$$f(x,y) = e^{-x^2} \cdot e^{-y^2} = \boxed{e^{-(x^2+y^2)}}$$

$$\iint_R f(x,y)\,dA = \int_0^{\frac{\pi}{4}}\int_1^2 e^{-r^2} \cdot r\, dr\, d\theta$$

$$\frac{d}{dr} e^{-r^2} = e^{-r^2} \cdot (-2r)$$

$$\int_0^{\frac{\pi}{4}} \int_1^2 r e^{-r^2} \, dr \, d\theta = \int_0^{\frac{\pi}{4}} \left[-\frac{1}{2} e^{-r^2} \right]_1^2 d\theta$$

$$= \int_0^{\frac{\pi}{4}} \left(-\frac{1}{2} e^{-2} + \frac{1}{2} e^{-1} \right) d\theta$$

$$= \frac{\pi}{4} \cdot \frac{1}{2} \left(\frac{1}{e} - \frac{1}{e^2} \right) = \frac{\pi}{8} \left(\frac{1}{e} - \frac{1}{e^2} \right)$$

$$\int_{r_1}^{r_2} \int_{\theta_1}^{\theta_2} f(r\cos\theta, r\sin\theta) \cdot r \, d\theta \, dr$$